Chapter 1: Basic Physics for Nuclear Medicine

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Nuclear Medicine Physics:
A Handbook for Teachers and Students

Objective:

To familiarize the student with the fundamental concepts of Physics for Nuclear Medicine

IAEA
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1.1 INTRODUCTION

1.1.1 Fundamental physical constants

- Avogadro’s number: $N_A = 6.022 \times 10^{23}$ atoms/mol
- Speed of light in vacuum: $c \approx 3 \times 10^{8}$ m/s
- Electron charge: $e = 1.602 \times 10^{-19}$ C
- Electron/positron rest mass: $m_e = 0.511$ MeV/c^2
- Proton rest mass: $m_p = 938.3$ MeV/c^2
- Neutron rest mass: $m_n = 939.6$ MeV/c^2

1.1 INTRODUCTION

1.1.1 Fundamental physical constants

- Atomic mass unit: $u = 931.5\ \mathrm{MeV}/c^2$
- Planck's constant: $h = 6.626 \times 10^{-34}\ \mathrm{J \cdot s}$
- Electric constant: (permittivity of vacuum): $\varepsilon_0 = 8.854 \times 10^{-12}\ \mathrm{C \cdot V^{-1} \cdot m^{-1}}$
- Magnetic constant: (permeability of vacuum) $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{V \cdot s \cdot A^{-1} \cdot m^{-1}}$
- Gravitation constant: $G = 6.672 \times 10^{-11}\ \mathrm{m^3 \cdot kg^{-1} \cdot s^{-2}}$

1.1 INTRODUCTION

1.1.2. Physical quantities and units

The SI system of units is founded on base units for seven physical quantities:

Quantity	SI unit
Length *l*	meter (m)
mass *m*	kilogram (kg)
time *t*	second (s)
electric current *I*	Ampère (A)
temperature *T*	kelvin (K)
amount of substance	mole (mol)
luminous intensity	candela (cd)

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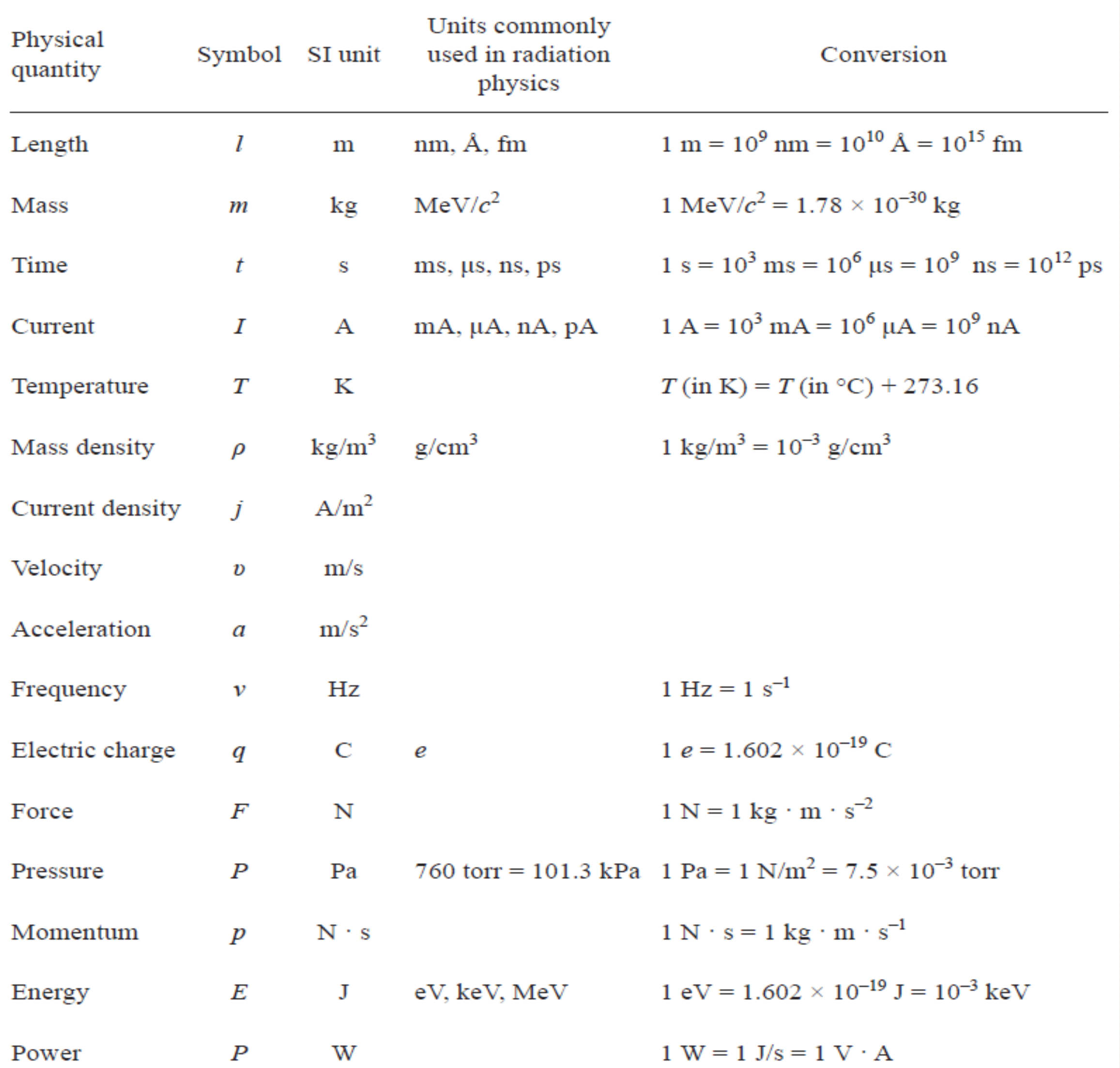
1.1 INTRODUCTION

1.1.2. Physical quantities and units

Basic quantities and several derived physical quantities and their units in SI units:



Physical quantity	Symbol	SI unit	Units commonly used in radiation physics	Conversion
Length	l	m	nm, Å, fm	1 m = 10^9 nm = 10^{10} Å = 10^{15} fm
Mass	m	kg	MeV/c^2	1 MeV/c^2 = 1.78×10^{-30} kg
Time	t	s	ms, μs, ns, ps	1 s = 10^3 ms = 10^6 μs = 10^9 ns = 10^{12} ps
Current	I	A	mA, μA, nA, pA	1 A = 10^3 mA = 10^6 μA = 10^9 nA
Temperature	T	K		T (in K) = T (in °C) + 273.16
Mass density	ρ	kg/m^3	g/cm^3	1 kg/m^3 = 10^{-3} g/cm^3
Current density	j	A/m^2		
Velocity	υ	m/s		
Acceleration	a	m/s^2		
Frequency	ν	Hz		1 Hz = 1 s^{-1}
Electric charge	q	C	e	1 e = 1.602×10^{-19} C
Force	F	N		1 N = 1 kg · m · s^{-2}
Pressure	P	Pa	760 torr = 101.3 kPa	1 Pa = 1 N/m^2 = 7.5×10^{-3} torr
Momentum	p	N · s		1 N · s = 1 kg · m · s^{-1}
Energy	E	J	eV, keV, MeV	1 eV = 1.602×10^{-19} J = 10^{-3} keV
Power	P	W		1 W = 1 J/s = 1 V · A

1.1 INTRODUCTION

1.1.4. Classification of ionizing radiation

☐ Ionizing radiation carries enough energy per quantum to remove an electron from an atom or molecule

- Introduces reactive and potentially damaging ion into the environment of the irradiated medium
- Can be categorized into two types:
  - Directly ionizing radiation
  - Indirectly ionizing radiation
- Both can traverse human tissue
  - Can be used in medicine for imaging & therapy

1.1 INTRODUCTION

1.1.5. Classification of indirectly ionizing photon radiation

- Consists of three main categories:
  - Ultraviolet: limited use in medicine
  - X ray: used in disease imaging and/or treatment
    - Emitted by orbital or accelerated electrons
  - γ ray: used in disease imaging and/or treatment
    - Emitted by the nucleus or particle decays
  - Difference between X and γ rays is based on the radiation’s origin

- The origin of these photons fall into 4 categories:
  - Characteristic (fluorescence) X rays
  - Bremsstrahlung X rays
  - From nuclear transitions
  - Annihilation quanta

1.1 INTRODUCTION

1.1.6. Characteristic X rays

- ☐ Orbital electrons inhabit atom’s minimal energy state
- ☐ An ionization or excitation process leads to an open vacancy
- ☐ An outer shell electron transitions to fill vacancy (~nsec)
- ☐ Liberated energy may be in the form of:
  - Characteristic photon (fluorescence)
    - Energy = initial state binding energy - final state binding energy
    - Photon energy is characteristic of the atom
  - Transferred to orbital electron that
    - Emitted with kinetic energy = transition energy - binding energy
    - Called an Auger electron

1.1 INTRODUCTION

1.1.7. Bremsstrahlung

- Translated from German as 'breaking radiation'
- Light charged particles (β^- & β^+) slowed down by interactions with other charged particles in matter (e.g. atomic nuclei)
- Kinetic energy loss converted to electromagnetic radiation
- Bremsstrahlung energy spectrum
  - Non-discrete (i.e. continuous)
  - Ranges: zero - kinetic energy of initial charged particle
- Central to modern imaging and therapeutic technology
  - Can be used to produce X rays from an electrical energy source

1.1 INTRODUCTION

1.1.8. Gamma rays

- Nuclear reaction or spontaneous nuclear decay may leave product (daughter) nucleus in excited state
- The nucleus can transition to a more stable state by emitting a γ ray
- Emitted photon energy is characteristic of nuclear energy transition
- γ ray energy typically > 100 keV & wavelengths < 0.1 Å

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1.1 INTRODUCTION

1.1.9. Annihilation quanta

☐ Positron results from:

- β^+ nuclear decay
- high energy photon interacts with nucleus or orbital electron electric field

☐ Positron kinetic energy (E_K) loss in absorber medium by Coulomb interactions:

- Collisional loss when interaction is with orbital electron
- Radiation loss (bremsstrahlung) when interaction is with the nucleus
- Final collision (after all E_K lost) with orbital electron (due to Coulomb attraction) called positron annihilation

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1.1 INTRODUCTION

1.1.9. Annihilation quanta

- ☐ During annihilation
  - Positron & electron disappear
  - Replaced by 2 oppositely directed annihilation quanta (photons)
  - Each has energy = 0.511 MeV
  - Conservation laws obeyed:
    - Electric charge, linear momentum, angular momentum, total energy

- ☐ In-flight annihilation
  - Annihilation can occur while positron still has kinetic energy
  - 2 quanta emitted
    - Not of identical energies
    - Do not necessarily move at 180º

1.1 INTRODUCTION

1.1.10. Radiation quantities and units

- ☐ Exposure: X
  - Ability of photons to ionize air
- ☐ Kerma: K (acronym for Kinetic Energy Released in MAtter)
  - Energy transferred to charged particles per unit mass of the absorber
  - Defined for indirectly ionizing radiation
- ☐ Dose (also referred to as absorbed dose):
  - Energy absorbed per unit mass of medium

1.1 INTRODUCTION

1.1.10. Radiation quantities and units

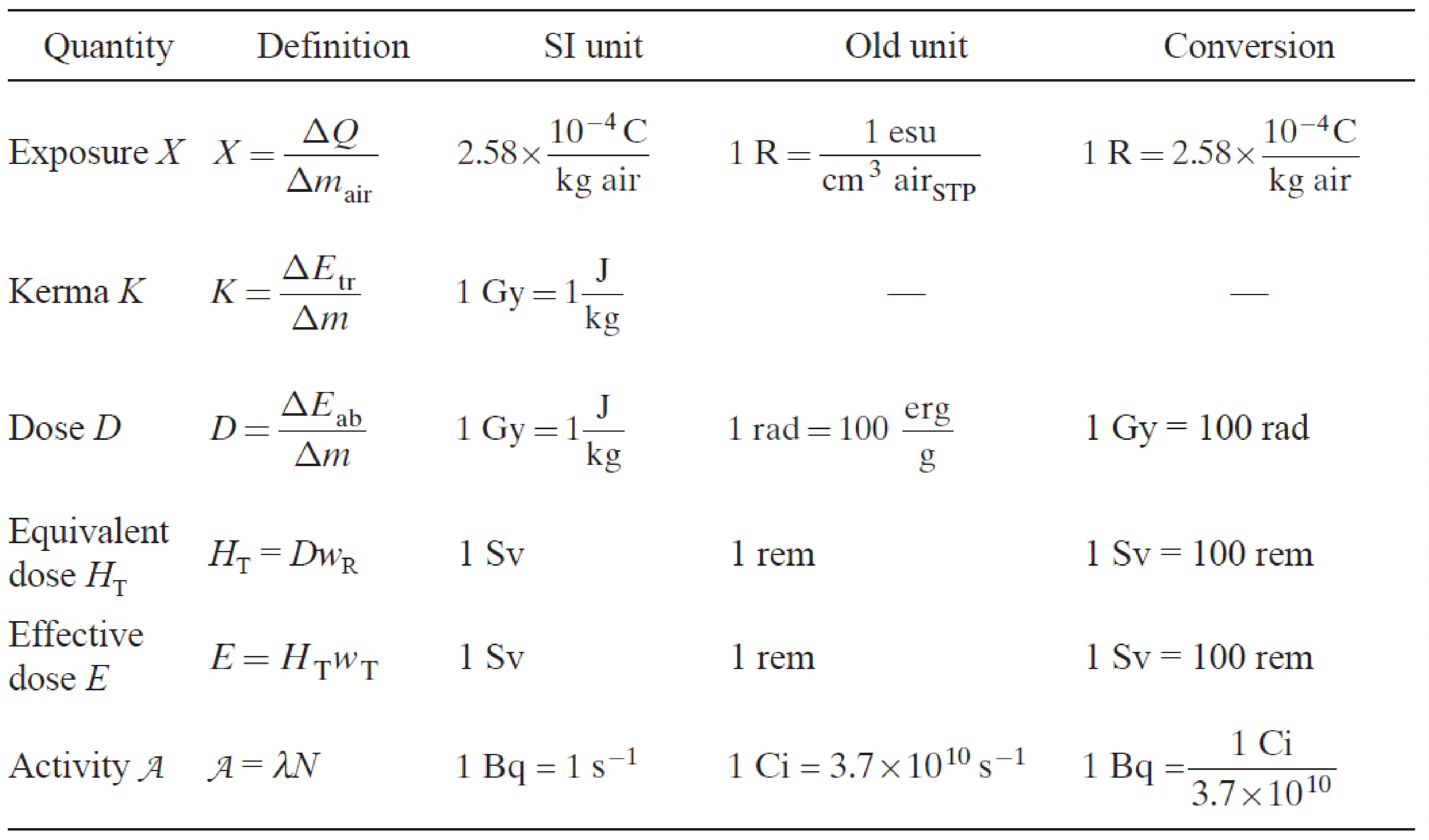
- Equivalent dose: H_T
  - Dose multiplied by radiation weighting factor w_R
  - When different types of radiation are present, H_T is the sum of all of the individual weighted contributions

- Effective dose: E
  - H_T multiplied by a tissue weighting factor w_T

- Activity: $\mathcal{A}$
  - Number of nuclear decays per unit time
  - Its SI unit, becquerel (Bq), corresponds to one decay per second

1.1 INTRODUCTION

1.1.10. Radiation quantities and units



Quantity	Definition	SI unit	Old unit	Conversion
Exposure X	$X=\dfrac{\Delta Q}{\Delta m_{\text{air}}}$	$2.58\times\dfrac{10^{-4}\ \text{C}}{\text{kg air}}$	$1\ \text{R}=\dfrac{1\ \text{esu}}{\text{cm}^3\ \text{air}_{\text{STP}}}$	$1\ \text{R}=2.58\times\dfrac{10^{-4}\text{C}}{\text{kg air}}$
Kerma K	$K=\dfrac{\Delta E_{\text{tr}}}{\Delta m}$	$1\ \text{Gy}=1\dfrac{\text{J}}{\text{kg}}$	—	—
Dose D	$D=\dfrac{\Delta E_{\text{ab}}}{\Delta m}$	$1\ \text{Gy}=1\dfrac{\text{J}}{\text{kg}}$	$1\ \text{rad}=100\ \dfrac{\text{erg}}{\text{g}}$	1 Gy = 100 rad
Equivalent dose H_{T}	$H_{\text{T}}=Dw_{\text{R}}$	1 Sv	1 rem	1 Sv = 100 rem
Effective dose E	$E=H_{\text{T}}w_{\text{T}}$	1 Sv	1 rem	1 Sv = 100 rem
Activity $\mathcal{A}$	$\mathcal{A}=\lambda N$	$1\ \text{Bq}=1\ \text{s}^{-1}$	$1\ \text{Ci}=3.7\times10^{10}\ \text{s}^{-1}$	$1\ \text{Bq}=\dfrac{1\ \text{Ci}}{3.7\times10^{10}}$

1.2 BASIC DEFINITIONS FOR ATOMIC STRUCTURE

- Constituent particles forming an atom are:
  - Proton
  - Neutron
  - Electron

known as nucleons

- $m_p/m_e = 1836$

- Atomic number: Z
  - Number of protons and number of electrons in an atom

- Atomic mass number: A
  - Number of nucleons in an atom = $Z + N$
  - Z = number of protons
  - N = number of neutrons

1.2 BASIC DEFINITIONS FOR ATOMIC STRUCTURE

☐ Atomic mass: m_a

- Mass of an atomic particle or molecule is expressed in atomic mass units u
- 1 u
  - 1/12th mass of carbon-12 atom
  - 931.5 MeV/c^2
- m_a < sum of masses of constituent particles: intrinsic energy associated with binding the particles (nucleons) in the nucleus

1.2 BASIC DEFINITIONS FOR ATOMIC STRUCTURE

- ☐ Molecular mole
  - For a given molecular compound, there are N_A molecules per mole of the compound
  - $N_A = 6.022 \text{ X} 10^{23} \text{ mol}^{-1}$

- ☐ The mass of a molecular mole will be the sum of the atomic mass numbers of the constituent atoms in the molecule

- ☐ For example:
  - 1 mole of water (H_2O) is 18 g of water
  - 1 mole of CO_2 is 44 g of carbon dioxide

1.2 BASIC DEFINITIONS FOR ATOMIC STRUCTURE

☐ For all elements the ratio $Z/A \approx 0.4\text{-}0.5$ with 1 notable exception:

- Hydrogen, for which $Z/A = 1$

☐ The ratio Z/A gradually decreases with increasing Z:

- From ~0.5 for low Z elements
- To ~0.4 for high Z elements

☐ For example:

- $Z/A = 0.50$ for ${}^{4}_{2}\mathrm{He}$
- $Z/A = 0.45$ for ${}^{60}_{27}\mathrm{Co}$
- $Z/A = 0.39$ for ${}^{235}_{92}\mathrm{U}$

1.3. BASIC DEFINITIONS FOR NUCLEAR STRUCTURE

- ☐ Most of the atomic mass is concentrated in the atomic nucleus

- ☐ Nucleus consists
  - Z protons
  - A - Z neutrons,

    where Z = atomic number and A = atomic mass

- ☐ Protons and neutrons
  - Commonly called nucleons
  - Bound to the nucleus with the strong force

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1.3. BASIC DEFINITIONS FOR NUCLEAR STRUCTURE

- Nuclear physics conventions
  - Designate a nucleus X as $^{A}_{Z}\mathrm{X}$

- For example*:*
  - Cobalt-60 nucleus
    - Z = 27 & A = 60 (i.e. 33 neutrons*)*
    - identified as: $^{60}_{27}\mathrm{Co}$
  - Radium-226
    - Z = 88 & A = 226 (i.e.138 neutrons)
    - identified as: $^{226}_{88}\mathrm{Ra}$

1.3. BASIC DEFINITIONS FOR NUCLEAR STRUCTURE

☐ Classifications

- Isotopes of an element
  - Atoms with same *Z*, but different number of neutrons (and *A*)
  - e.g. $^{59}_{27}\mathrm{Co}$ $^{60}_{27}\mathrm{Co}$
  - ‘Nuclide’ refers to an atomic species, defined by its makeup of protons, neutrons, and energy state
  - ‘Isotope’ refers to various atomic forms of a given chemical element
- Isobars
  - Common atomic mass number *A*
  - e.g. ^{60}Co and ^{60}Ni

1.3. BASIC DEFINITIONS FOR NUCLEAR STRUCTURE

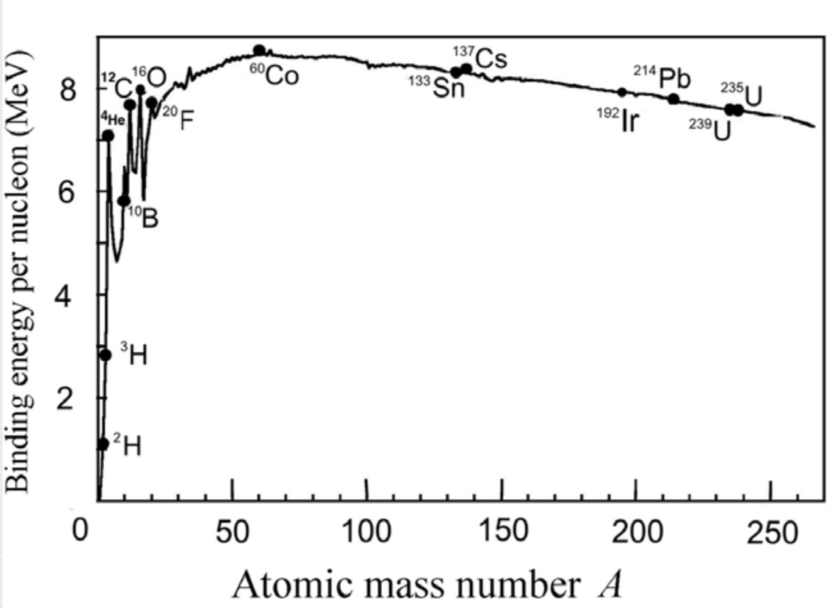
☐ Classifications

- Isotones
  - Common number of neutrons
  - e.g. ^{3}H (tritium) and ^{4}He
- Isomeric (metastable) state
  - Excited nuclear state that exists for some time
  - e.g ^{99m}Tc is an isomeric state of ^{99}Tc

1.3. BASIC DEFINITIONS FOR NUCLEAR STRUCTURE

1.3.2. Nuclear binding energy

- E_B/A (Binding energy per nucleon)
  - Varies with *A*
  - ~8 MeV/nucleon
  - Rises rapidly at small *A*
  - Broad maximum
    - ~ 8.7 MeV/nucleon
    - $A \approx 60$
  - Gradual decrease at large *A*
  - Larger value implies atom more stable
  - Most stable nuclei have $A \approx 60$
    - Fe, Co, Ni



1.3. BASIC DEFINITIONS FOR NUCLEAR STRUCTURE

1.3.3. Nuclear fusion and fission

- ☐ E_B/A vs. A curve suggests 2 methods for mass to energy conversion:

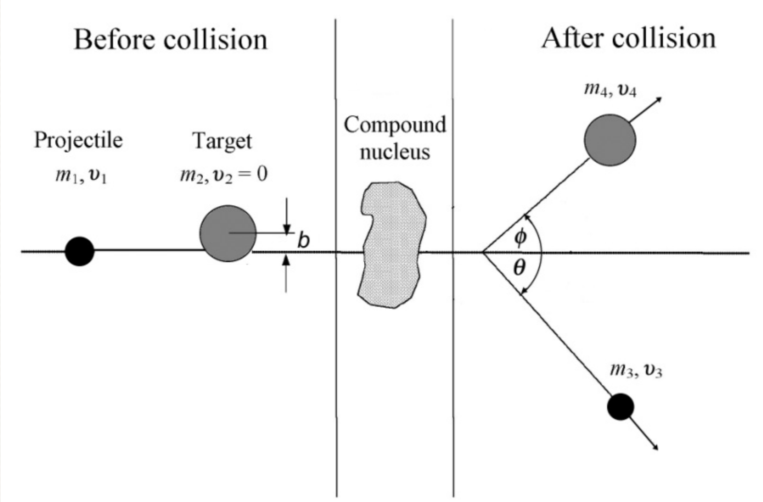
  1) Fusion of low *A* nuclei
     - Creates a more massive nucleus
     - Releases energy
     - Presently, controlled fusion for energy production not successful in net energy generation
     - Remains active field of research
  2) Fission of large *A* nuclei
     - Bombardment of large mass elements (e.g. ^{235}U) by thermal neutrons will create 2 more stable nuclei with lower mass
     - Process transforms some mass into kinetic energy
     - Fission reactors are important means of production of electrical power

1.3. BASIC DEFINITIONS FOR NUCLEAR STRUCTURE

1.3.4. Two-particle collisions and nuclear reactions

☐ 2 particle collision

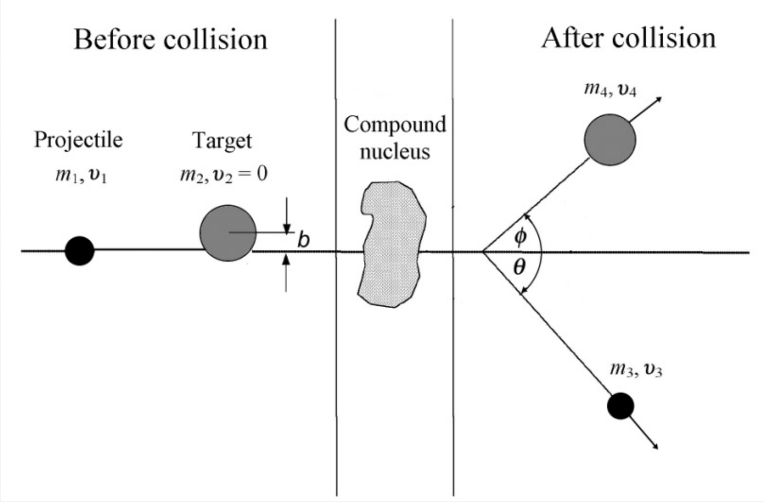
- Projectile: mass m_1, velocity υ_1, kinetic energy $(E_K)_1$
- Stationary target : mass m_2 & $\upsilon_2 = 0$
- Results in intermediate compound
- Decays into 2 reaction products: (m_3, υ_3) and (m_4, υ_4)
- Cross-section (probability for collision) & collision outcome depends on:
  - Projectile mass, charge, velocity, kinetic energy
  - Stationary target mass, charge



1.3. BASIC DEFINITIONS FOR NUCLEAR STRUCTURE

1.3.4. Two-particle collisions and nuclear reactions

- ☐ Projectile + target collision: most general case
  - Results in intermediate compound
  - Decays into 2 reaction products:
    - m_3 ejected with υ_3 at θ to incident projectile direction
    - m_4 ejected with υ_4 at ϕ to incident projectile direction



1.3. BASIC DEFINITIONS FOR NUCLEAR STRUCTURE

1.3.4. Two-particle collisions and nuclear reactions

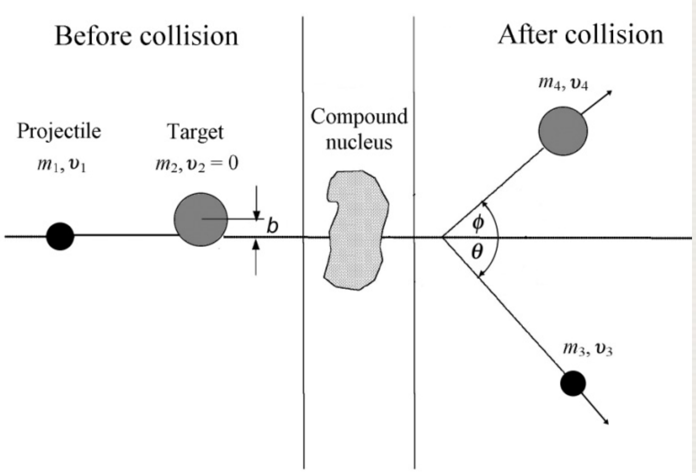
- Two-particle collisions classified into 3 categories:

1) Elastic

   - Products after identical to products before collision
     - $m_3 = m_1$ and $m_4 = m_2$
     - Total kinetic energy & momentum before & after collision are equal

2) Inelastic projectile scattering

   - Products after identical to products before collision
   - Incident projectile transfers portion of its E_K to target as E_K + intrinsic excitation energy E^*

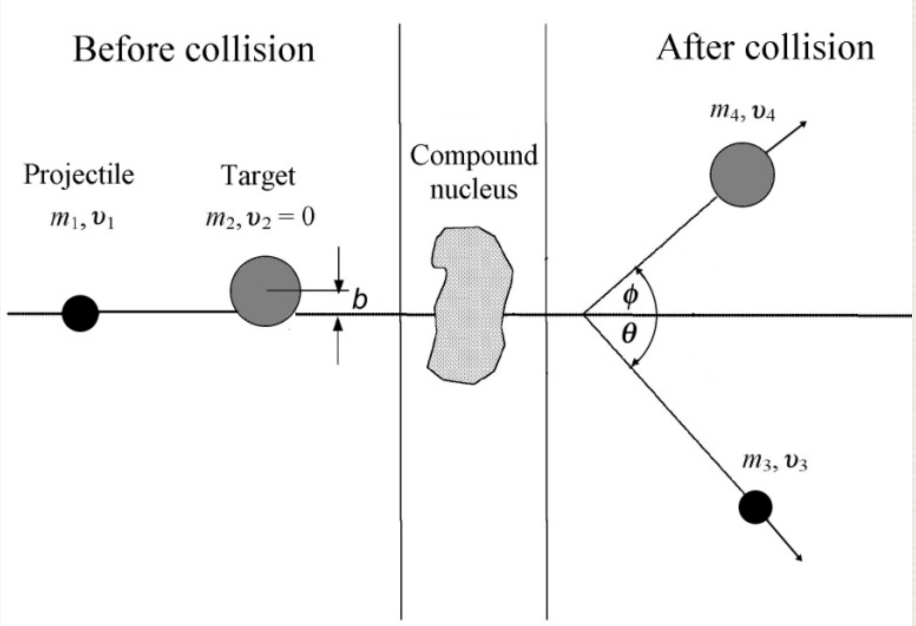


1.3. BASIC DEFINITIONS FOR NUCLEAR STRUCTURE

1.3.4. Two-particle collisions and nuclear reactions

3) Nuclear reaction

- 2 products $m_3 + m_4$, with new Z
- Physical quantities must be conserved
  - Charge
  - Linear momentum
  - Mass–energy
  - Sum of Z's & sum of A's



1.3. BASIC DEFINITIONS FOR NUCLEAR STRUCTURE

1.3.4. Two-particle collisions and nuclear reactions

- ❑ $(E_\mathrm{K})_\mathrm{thr}$ is calculated from the relativistic invariant = smallest value of projectile E_K at which reaction will take place:

$$(E_\mathrm{K})_\mathrm{thr} = \frac{(m_3c^2 + m_4c^2)^2 - (m_1c^2 + m_2c^2)^2}{2m_2c^2} \approx -Q\left(1 + \frac{m_1}{m_2}\right)$$

- ❑ m_1c^2, m_2c^2, m_3c^2 and m_4c^2 are rest energies of projectile m_1, target m_2 & reaction products m_3 and m_4, respectively

1.4. RADIOACTIVITY

1.4.1. Decay of radioactive parent into a stable or unstable daughter

- ☐ Decay of radioactive parent P into stable daughter D, with decay constant λ_P:

$$\mathrm{P} \xrightarrow{\lambda_{\mathrm{P}}} \mathrm{D}$$

- ☐ Rate of depletion of the number of radioactive parent nuclei, $N_P(t)$, is equal to the activity $\mathcal{A}_P(t)$ at time t:

$$\frac{\mathrm{d}N_{\mathrm{P}}(t)}{\mathrm{d}t} = -\mathcal{A}_{\mathrm{P}}(t) = -\lambda_{\mathrm{P}} N_{\mathrm{P}}(t)$$

- ☐ Fundamental differential equation for $N_P(t)$ can be rewritten in integral form:

$$\int_{N_{\mathrm{P}}(0)}^{N_{\mathrm{P}}(t)} \frac{\mathrm{d}N_{\mathrm{P}}(t)}{N_{\mathrm{P}}} = -\int_0^t \lambda_{\mathrm{P}} \mathrm{d}t$$

  - $N_P(0)$ is the initial number of parent nuclei at time $t = 0$

1.4. RADIOACTIVITY

1.4.1. Decay of radioactive parent into a stable or unstable daughter

- Number of radioactive parent nuclei as a function of time *t*, assuming that λ_P is constant, is:

$$N_P(t) = N_P(0)\mathrm{e}^{-\lambda_P t}$$

- Activity of the radioactive parent $\mathcal{A}_P(t)$ as a function of time *t*:

$$\mathcal{A}_P(t) = \lambda_P N_P(t) = \lambda_P N_P(0)\mathrm{e}^{-\lambda_P t} = \mathcal{A}_P(0)\mathrm{e}^{-\lambda_P t}$$

  - where $\mathcal{A}_P(0)$ is the initial activity at time $t = 0$

- Decay law applies to all radioactive nuclides irrespective of decay mode

1.4. RADIOACTIVITY

1.4.1. Decay of radioactive parent into a stable or unstable daughter

- Half-life, $(T_{1/2})_\mathrm{P}$, of radioactive parent P is the time during which the number of radioactive parent nuclei decays from the initial value, $N_\mathrm{P}(0)$, at time $t = 0$ to half the initial value ($\mathcal{A}_\mathrm{P}(t)$ also decreases to half of its initial value)

$$N_\mathrm{P}\left[t = (T_{1/2})_\mathrm{P}\right] = \frac{1}{2} N_P(0) = N_\mathrm{P}(0)\mathrm{e}^{-\lambda_\mathrm{P}(T_{1/2})_\mathrm{P}}$$

$$\mathcal{A}_\mathrm{P}\left[t = (T_{1/2})_\mathrm{P}\right] = \frac{1}{2} \mathrm{A}_\mathrm{P}(0) = \mathrm{A}_\mathrm{P}(0)\mathrm{e}^{-\lambda_\mathrm{P}(T_{1/2})_\mathrm{P}}$$

- λ_P & $(T_{1/2})_\mathrm{P}$ are related as follows:

$$\lambda_\mathrm{P} = \frac{\ln 2}{(T_{1/2})_\mathrm{P}} = \frac{0.693}{(T_{1/2})_\mathrm{P}}$$

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1.4. RADIOACTIVITY

1.4.1. Decay of radioactive parent into a stable or unstable daughter

- Mean (average) life τ_P of a radioactive parent P is the time during which the number N_P of radioactive nuclei or its activity $\mathcal{A}_\text{P}$ falls to 1/e = 0.368 (or 36.8%) of $N_\text{P}(0)$ or of $\mathcal{A}_\text{P}(0)$, respectively

$$N_\text{P}\left(t=\tau_\text{P}\right)=\frac{1}{\text{e}}N_\text{P}(0)=0.368N_\text{P}(0)=N_\text{P}(0)\text{e}^{-\lambda_\text{P}\tau_\text{P}}$$

$$\mathcal{A}_\text{P}\left(t=\tau_\text{P}\right)=\frac{1}{\text{e}}\mathcal{A}_\text{P}(0)=0.368\mathcal{A}_\text{P}(0)=\mathcal{A}_\text{P}(0)\text{e}^{-\lambda_\text{P}\tau_\text{P}}$$

- λ_P & $(T_{1/2})_\text{P}$ are related as follows:

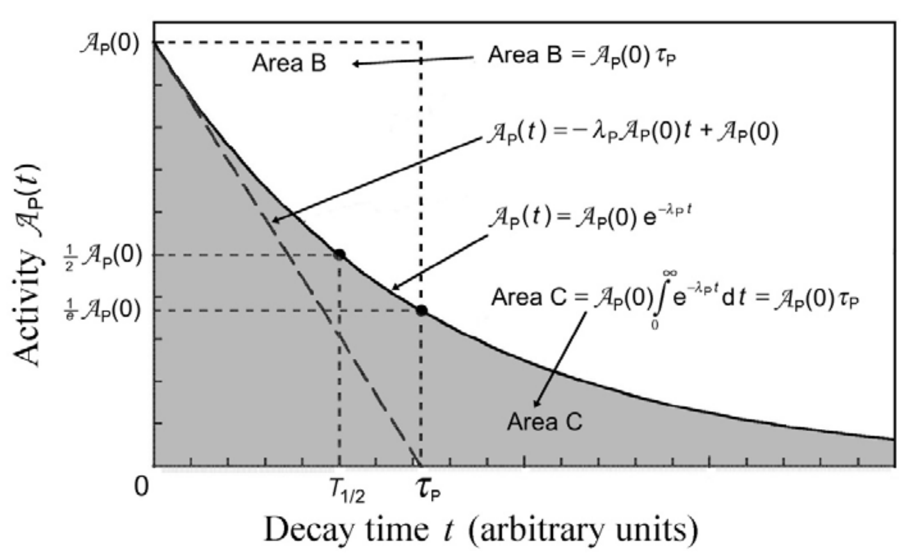
$$\lambda_\text{P}=\frac{\ln 2}{(T_{1/2})_\text{P}}=\frac{1}{\tau_\text{P}} \qquad \text{and}$$

$$\tau_\text{P}=\frac{(T_{1/2})_\text{P}}{\ln 2}=1.44(T_{1/2})_\text{P}$$

1.4. RADIOACTIVITY

1.4.1. Decay of radioactive parent into a stable or unstable daughter

- Activity $\mathcal{A}_{\mathrm{P}}(t)$ plotted against time t for a simple decay of a radioactive parent P to stable or unstable daughter D:



Illustrates:

- Concept of $(T_{1/2})_{\mathrm{P}}$
- Concept of τ_{P}
- Exponential decay
- Area under curve from $t = 0$ to $t = \infty$ is equal to $\mathcal{A}_{\mathrm{P}}(t) \times \tau_{\mathrm{P}}$
- Slope of tangent to decay curve at $t = 0$ is $\lambda_{\mathrm{P}} \times \mathcal{A}_{\mathrm{P}}(0)$
- Abscissa intercept at $t = \tau_{\mathrm{P}}$

1.4. RADIOACTIVITY

1.4.2. Radioactive series decay

- Decay of radioactive parent P into unstable daughter D which in turn decays into granddaughter G:

$$\mathrm{P} \xrightarrow{\lambda_\mathrm{P}} \mathrm{D} \xrightarrow{\lambda_\mathrm{D}} \mathrm{G}$$

- Rate of change dN_P/dt in the number of daughter nuclei D equals to supply of new daughter nuclei through decay of P given as $\lambda_\mathrm{P} N_\mathrm{P}(t)$ & the loss of daughter nuclei D from the decay of D to G given as $-\lambda_\mathrm{D} N_\mathrm{D}(t)$

$$\frac{\mathrm{d}N_\mathrm{D}(t)}{\mathrm{d}t} = \lambda_\mathrm{P} N_\mathrm{P}(t) - \lambda_\mathrm{D} N_\mathrm{D}(t) = \lambda_\mathrm{P} N_\mathrm{P}(0)\mathrm{e}^{-\lambda_\mathrm{P} t} - \lambda_\mathrm{D} N_\mathrm{D}(t)$$

1.4. RADIOACTIVITY

1.4.2. Radioactive series decay

☐ Number of daughter nuclei is, assuming no daughter D nuclei present initially, i.e. $N_D(0) = 0$:

$$N_{\mathrm{D}}(t) = N_{\mathrm{P}}(0)\frac{\lambda_{\mathrm{P}}}{\lambda_{\mathrm{D}} - \lambda_{\mathrm{P}}}\left[\mathrm{e}^{-\lambda_{\mathrm{P}} t} - \mathrm{e}^{-\lambda_{\mathrm{D}} t}\right]$$

☐ Activity of the daughter nuclei is:

$$\mathcal{A}_{\mathrm{D}}(t) = \frac{N_{\mathrm{P}}(0)\lambda_{\mathrm{P}}\lambda_{\mathrm{D}}}{\lambda_{\mathrm{D}} - \lambda_{\mathrm{P}}}\left[\mathrm{e}^{-\lambda_{\mathrm{P}} t} - \mathrm{e}^{-\lambda_{\mathrm{D}} t}\right] = \mathcal{A}_{\mathrm{P}}(0)\frac{\lambda_{\mathrm{D}}}{\lambda_{\mathrm{D}} - \lambda_{\mathrm{P}}}\left[\mathrm{e}^{-\lambda_{\mathrm{P}} t} - \mathrm{e}^{-\lambda_{\mathrm{D}} t}\right] =$$

$$= \mathcal{A}_{\mathrm{P}}(0)\frac{1}{1 - \dfrac{\lambda_{\mathrm{P}}}{\lambda_{\mathrm{D}}}}\left[\mathrm{e}^{-\lambda_{\mathrm{P}} t} - \mathrm{e}^{-\lambda_{\mathrm{D}} t}\right] = \mathcal{A}_{\mathrm{P}}(t)\frac{\lambda_{\mathrm{D}}}{\lambda_{\mathrm{D}} - \lambda_{\mathrm{P}}}\left[1 - \mathrm{e}^{-(\lambda_{\mathrm{D}} - \lambda_{\mathrm{P}})t}\right]$$

- $\mathcal{A}_D(t)$ = activity at time t of daughter = $\lambda_D N_D(t)$
- $\mathcal{A}_P(0)$ = initial activity of parent at time $t = 0$
- $\mathcal{A}_P(t)$ = activity of parent at time $t = \lambda_P N_P(t)$

1.4. RADIOACTIVITY

1.4.2. Radioactive series decay

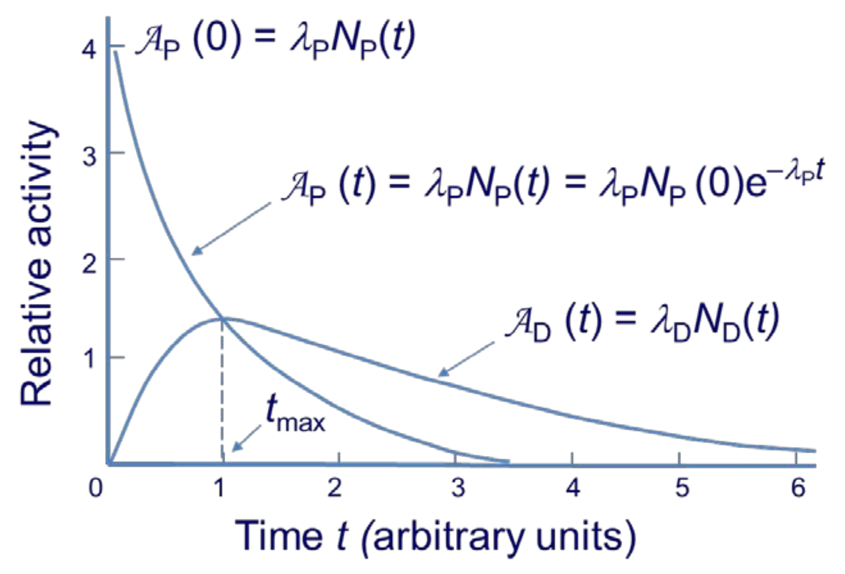
☐ Daughter activity $\mathcal{A}_D(t)$ *vs* time

- For the case $\mathcal{A}_D(0) = 0$
- Daughter activity initially rises with time t
- Reaches maximum at characteristic time $t = (t_{max})_D$
- Diminishes to reach 0 at $t = \infty$

$$(t_{max})_D = \frac{\ln \dfrac{\lambda_P}{\lambda_D}}{\lambda_P - \lambda_D}$$

Parent and daughter activities against time for

$$P \xrightarrow{\lambda_P} D \xrightarrow{\lambda_D} G$$



1.4. RADIOACTIVITY

1.4.3. Equilibrium in parent — daughter activities

❑ Radioactive equilibrium

- Occurs in many P → D → G relationships
- Parent & daughter activities reach constant ratio after a certain time *t*

❑ $\mathcal{A}_D(t)/\mathcal{A}_P(t)$ ratio behaviour:

$$\frac{\mathcal{A}_D(t)}{\mathcal{A}_P(t)} = \frac{\lambda_D}{\lambda_D - \lambda_P}\left[1 - e^{-(\lambda_D - \lambda_P)t}\right] = \frac{1}{1 - \frac{\lambda_P}{\lambda_D}}\left[1 - e^{-(\lambda_D - \lambda_P)t}\right]$$

1.4. RADIOACTIVITY

1.4.4. Production of radionuclides (nuclear activation)

☐ Nuclear activation

- Bombardment of a stable nuclide with a suitable energetic particle or high energy photons to induce a nuclear transformation
  - Neutrons from nuclear reactors for neutron activation
  - Protons from cyclotrons or synchrotrons for proton activation
  - X rays from high energy linear accelerators for nuclear photoactivation

1.4. RADIOACTIVITY

1.4.4. Production of radionuclides (nuclear activation)

☐ Neutron activation important in production of radionuclides used for

- External beam radiotherapy
- Brachytherapy
- Therapeutic nuclear medicine
- Nuclear medicine imaging (molecular imaging)

☐ Proton activation important in production of positron emitters used in

- Positron emission tomography (PET) imaging

☐ Nuclear photoactivation important from a radiation protection point of view

- Components of high energy radiotherapy machines become activated during patient treatment
  - Potential radiation risk to staff using equipment

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1.4. RADIOACTIVITY

1.4.5. Modes of radioactive decay

- ☐ Nucleons are bound together to form nucleus by strong nuclear force
  - At least two orders of magnitude larger than proton–proton Coulomb repulsive force
  - Extremely short range (a few femtometres)

- ☐ A delicate equilibrium between number of protons and number of neutrons must exist to bind the nucleons into a stable nucleus
  - Configurations to form stable nuclei
    - For low *A* nuclei → $Z = N$
    - For $A \geq 40$ → $N > Z$ (in order to overcome proton-proton Coulomb repulsion)

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1.4. RADIOACTIVITY

1.4.5. Modes of radioactive decay

☐ If there is no proton-neutron optimal equilibrium:

- Nucleus is unstable (radioactive)
- Nucleus decays with a specific decay constant λ into more stable configuration that may also be unstable and decay further, forming a decay chain that eventually ends with a stable nuclide

1.4. RADIOACTIVITY

1.4.5. Modes of radioactive decay

- Radioactive decay is a process by which unstable (radioactive) nuclei reach a more stable configuration

- Radioactive decay processes
  - Medically important
    - Alpha (α) decay
    - Beta (β) decay
      - Beta plus decay
      - Beta minus decay
      - Electron capture
    - Gamma (γ) decay
      - Pure gamma decay
      - Internal conversion
  - Less important
    - Spontaneous fission

1.4. RADIOACTIVITY

1.4.5. Modes of radioactive decay

- ☐ Neutron-rich nuclides have excess number of neutrons
- ☐ Proton-rich nuclides have excess number of protons
- ☐ Decays:
  - Slight Proton–neutron imbalance:
    - Proton into a neutron in β^+ decay
    - Neutron into a proton in β^- decay
  - Large proton–neutron imbalance:
    - α particles in α decay OR protons in proton emission decay
    - Neutrons in neutron emission decay
  - Very large *A* nuclides ($A > 230$)
    - Spontaneous fission competing with α decay

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1.4. RADIOACTIVITY

1.4.5. Modes of radioactive decay

☐ Excited nuclei decay to ground state via γ decay

- Most of these occur immediately upon excited state production by α or β decay

- A few have delayed decays governed by their own decay constants
  - Referred to as metastable states (e.g. ^{99m}Tc)

1.4. RADIOACTIVITY

1.4.5. Modes of radioactive decay

- Nuclear transformations are usually accompanied by emission of energetic particles (charged particles, neutral particles, photons, neutrinos)

Radioactive decay	Emitted particles
Alpha decay	α particle
Beta plus decay	β^+ particle (positron), neutrino
Beta minus decay	β^- particle (electron), antineutrino
Electron capture	Neutrino
Pure gamma decay	Photon
Internal conversion	Orbital electron
Spontaneous fission	Fission products, neutrons, heavier nuclei
Neutron emission decay	Neutron
Proton emission decay	Proton

1.4. RADIOACTIVITY

1.4.5. Modes of radioactive decay

☐ In each nuclear transformation a number of physical quantities must be conserved

☐ The most important conserved physical quantities are:

- Total energy
- Momentum
- Charge
- Atomic number
- Atomic mass number (number of nucleons)

1.4. RADIOACTIVITY

1.4.5. Modes of radioactive decay

- ☐ Total energy of particles released by the transformation process is equal to the net decrease in the rest energy of the neutral atom, from parent P to daughter D

- ☐ Decay energy (*Q* value) is given as:

$$Q = \left\{ M(\mathrm{P}) - \left[M(\mathrm{D}) + m \right] \right\} \cdot c^2$$

M(P), *M*(D), and *m* are the nuclear rest masses of the parent, daughter and emitted particles, respectively (in unified atomic mass units u)

- ☐ Radioactive decay energetically possible if *Q* > 0, thus
  - Spontaneous radioactive decay processes are exoergic or exothermic
  - Energy equivalent of *Q* is shared as E_K between emitted particles & the daughter product
    - Usually *M*(D) >> *m* → E_K of daughter usually negligibly small

1.4. RADIOACTIVITY

1.4.6. Alpha decay

- ☐ Alpha decay is a nuclear transformation in which:
  - Energetic α particle, ^{4}He nucleus ($^{4}\mathrm{He}^{2+}$) is emitted
  - Atomic number *Z* of the parent decreases by 2
  - Atomic mass number A of the parent decreases by 4

$$ {}^{A}_{Z}\mathrm{P} \rightarrow {}^{A-4}_{Z-2}\mathrm{D} + {}^{4}_{2}\mathrm{He}^{2+} = {}^{A-4}_{Z-2}\mathrm{D} + \alpha $$

- ☐ Naturally occurring α's
  - E_{K} : 4-9 MeV
  - Range in air: 1-10 cm
  - Range in tissue: 10 - 100 μm

- ☐ Examples:

$$ {}^{226}_{88}\mathrm{Ra} \xrightarrow[T_{1/2}=1602\,\mathrm{y}]{} {}^{222}_{86}\mathrm{Rn} + \alpha $$

$$ {}^{222}_{86}\mathrm{Rn} \xrightarrow[T_{1/2}=3.82\,\mathrm{d}]{} {}^{218}_{84}\mathrm{Po} + \alpha $$

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1.4. RADIOACTIVITY

1.4.7. Beta minus decay

❑ Beta minus (β^-) decay :

- Neutron-rich parent nucleus P
  - Transforms neutron into proton: $\mathrm{n} \rightarrow \mathrm{p} + \mathrm{e}^- + \bar{\nu}_\mathrm{e}$
  - Ejects e^- & antineutrino, which share available energy
- $Z_\mathrm{D} = Z_\mathrm{P} + 1$
- $A_\mathrm{D} = A_\mathrm{P}$
- Daughter D isobar of parent P

$$ {}^{A}_{Z}\mathrm{P} \rightarrow {}^{A}_{Z+1}\mathrm{D} + \mathrm{e}^- + \bar{\nu}_\mathrm{e} $$

- Example of β^- decay

$$ {}^{60}_{27}\mathrm{Co} \xrightarrow[T_{1/2}=5.26\,\mathrm{y}]{} {}^{60}_{28}\mathrm{Ni}^* + \mathrm{e}^- + \bar{\nu}_\mathrm{e} $$

1.4. RADIOACTIVITY

1.4.8. Beta plus decay

☐ Beta plus (β^+) decay:

- Proton-rich parent nucleus P
  - transforms a proton into a neutron

$$p \rightarrow n + e^+ + \nu_e$$

  - Ejects e^+ & ν_e, which share available energy
- $Z_D = Z_P - 1$
- $A_D = A_P$
- Daughter D isobar of parent P

$$^A_Z P \rightarrow \, ^{A}_{Z-1}D + e^+ + \nu_e$$

1.4. RADIOACTIVITY

1.4.8. Beta plus decay

- ❑ Radionuclides undergoing β^+ decay often called positron emitters
  - Used in medicine for PET functional imaging
  - Most common PET tracer is fluorodeoxyglucose (FDG) labelled with ^{18}F

- ❑ Example of β^+ decay

$$^{18}_{9}\mathrm{F} \xrightarrow[T_{1/2}=110\,\mathrm{min}]{} {}^{18}_{8}\mathrm{O} + e^{+} + \nu_{e}$$

1.4. RADIOACTIVITY

1.4.9. Electron capture

- Electron capture is a nuclear transformation in which:
  - Nucleus captures an atomic orbital electron (usually K shell)

$$\mathrm{p} + \mathrm{e}^{-} = \mathrm{n} + \nu_{\mathrm{e}}$$

  - $Z_{\mathrm{D}} = Z_{\mathrm{P}} - 1$
  - $A_{\mathrm{D}} = A_{\mathrm{P}}$
  - Daughter D isobar of parent P

$$^{A}_{Z}\mathrm{P} + \mathrm{e}^{-} \rightarrow \, ^{A}_{Z-1}\mathrm{D} + \nu_{\mathrm{e}}$$

- Example of e^{-} capture

$$^{125}_{53}\mathrm{I} + \mathrm{e}^{-} \xrightarrow[T_{1/2}=60\mathrm{d}]{} \, ^{125}_{52}\mathrm{Te}^{*} + \nu_{\mathrm{e}}$$

  - ^{125}Te* is the excited state of ^{125}Te
  - decays to ^{125}Te ground state by γ decay & internal conversion

1.4. RADIOACTIVITY

1.4.10. Gamma decay and internal conversion

☐ α, β^-, β^+ and electron capture, may produce daughter (D) nucleus in excited state

- Full amount of the decay energy available not expended
- Will reach ground (de-excite) state by:
  - Emitting excitation energy as one or more γ
  - Internal conversion
    - Transfer of excitation energy to atomic orbital electrons (usually K shell)
    - Vacancy in shell filled by higher orbital electron
    - Resulting in characteristic X rays and/or Auger electrons

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1.4. RADIOACTIVITY

1.4.10. Gamma decay and internal conversion

- ☐ In most α & β decays de-excitation is instantaneous
  - Thus, we refer to emitted γ's as if produced by parent
  - e.g. ^{60}Co γ rays
- ☐ Sometimes, D de-excites with time delay
  - Excited state of D is referred to as a metastable state
  - De-excitation called isomeric transition
  - e.g. ^{99m}Tc

1.4. RADIOACTIVITY

1.4.10. Gamma decay and internal conversion

☐ γ decay

$$^{A}_{Z}\mathrm{X}^{*} \rightarrow {}^{A}_{Z}\mathrm{X} + \gamma$$

- $^{A}_{Z}\mathrm{X}^{*}$ = excited stated of $^{A}_{Z}\mathrm{X}$

Example:

$$^{60}_{27}\mathrm{Co} \rightarrow {}^{60}_{28}\mathrm{Ni}^{*} + \mathrm{e}^{-} + \overline{\nu}_{\mathrm{e}}$$

$$^{60}_{28}\mathrm{Ni}^{*} \rightarrow {}^{60}_{28}\mathrm{Ni} + \gamma_{1} + \gamma_{2}$$

- Where $E_{\gamma 1}$=1.17 MeV & $E_{\gamma 2}$=1.33MeV

1.4. RADIOACTIVITY

1.4.10. Gamma decay and internal conversion

☐ Internal conversion

$${}^{A}_{Z}\mathrm{X}^{*} \rightarrow {}^{A}_{Z}\mathrm{X}^{+} + \mathrm{e}^{-} \rightarrow {}^{A}_{Z}\mathrm{X}$$

- ${}^{A}_{Z}\mathrm{X}^{+}$ = singly ionized state of ${}^{A}_{Z}\mathrm{X}$

- Example:

$${}^{125}_{53}\mathrm{I} + \mathrm{e}^{-} \rightarrow {}^{125}_{52}\mathrm{Te}^{*} + \nu_{\mathrm{e}}$$

$${}^{125}_{52}\mathrm{Te}^{*} \rightarrow {}^{125}_{52}\mathrm{Te} + \gamma \ (7\%)$$

$$\text{or} \quad {}^{125}_{52}\mathrm{Te}^{*} \rightarrow {}^{125}_{52}\mathrm{Te} + \mathrm{e}^{-} (\text{internal conversion } 93\%)$$

1.4. RADIOACTIVITY

1.4.11 Characteristic (fluorescence) X rays and Auger electrons

- ☐ A large number of radionuclides used in nuclear medicine (e.g. ^{99m}Tc, ^{123}I, ^{201}Tl, ^{64}Cu) decay by electron capture and/or internal conversion

- ☐ Both processes leave the atom with a vacancy in an inner atomic shell
  - Most commonly the K shell
  - Inner shell vacancy filled by electron from higher level atomic shell
  - Binding energy difference between the two shells is emitted as
    - Characteristic X ray (fluorescence photon)
    - Or transferred to higher shell orbital electron
      - Then emitted from atom as Auger electron with E_K equal to transferred energy minus the binding energy of the emitted Auger electron

1.5. ELECTRON INTERACTIONS WITH MATTER

☐ Energetic charged particles (e.g. e^- or e^+) undergo Coulomb interactions with absorber atoms, i.e., with:

- Atomic orbital electrons
  - Ionization loss
- Atomic nuclei
  - Radiation loss

☐ Through these collisions the electrons may:

- Lose their kinetic energy (collision and radiation loss)
- Change direction of motion (scattering)

1.5. ELECTRON INTERACTIONS WITH MATTER

- ☐ Interactions between the charged particle and absorber atom is characterized by a specific cross-section (probability) σ

- ☐ Energy loss depends on
  - Particle properties (mass, charge, velocity & energy)
  - Absorber properties (density & Z)

1.5. ELECTRON INTERACTIONS WITH MATTER

- ☐ Gradual loss of energy of charged particle described by stopping power

- ☐ Two classes of stopping power known
  - Collision stopping power s_{col} from interaction with orbital electrons of absorber
  - Radiation stopping power s_{rad} from interaction with nuclei of absorber

- ☐ Total stopping power: $s_{tot} = s_{col} + s_{rad}$

1.5. ELECTRON INTERACTIONS WITH MATTER

1.5.1. Electron–orbital interactions

- ☐ Inelastic collisions between the incident electron and an orbital electron are Coulomb interactions that result in:
  - Atomic ionization:
    - Ejection of the orbital electron from the absorber atom
    - Absorber atom becomes ion
  - Atomic excitation:
    - Transfer of an atomic orbital electron from one allowed orbit (shell) to a higher level allowed orbit
    - Absorber atom becomes excited atom
- ☐ Atomic excitations & ionizations result in collision energy losses and are characterized by collision (ionization) stopping power

1.5. ELECTRON INTERACTIONS WITH MATTER

1.5.2. Electron–nucleus interactions

☐ Coulomb interaction between the incident electron and an absorber nucleus results in:

- Electron scattering and no energy loss (elastic collision): characterized by angular scattering power

- Electron scattering and some loss of kinetic energy in the form of bremsstrahlung (radiation loss): characterized by radiation stopping power

1.6. PHOTON INTERACTIONS WITH MATTER

1.6.1. Exponential absorption of photon beam in absorber

- ☐ The most important parameter used for characterization of X or γ ray penetration into absorbing media is the linear attenuation coefficient μ

- ☐ Linear attenuation coefficient μ depends on:
  - Energy $h\nu$ of photon
  - Z of the absorber

- ☐ Linear attenuation coefficient may be described as the probability per unit path length that a photon will have an interaction with the absorber

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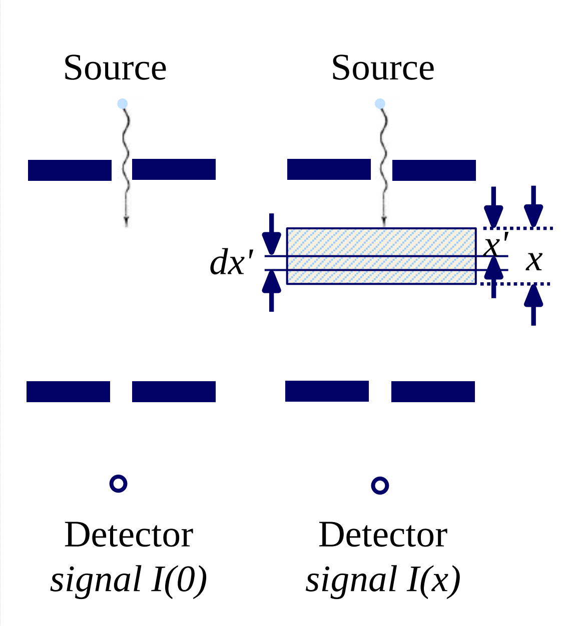
1.6. PHOTON INTERACTIONS WITH MATTER

1.6.1. Exponential absorption of photon beam in absorber

- Attenuation coefficient, μ, is determined experimentally by:
  - Aiming narrowly collimated mono-energetic photon beam ($E = h\nu$)
  - Placing absorber material of varying thicknesses x between photon source and detector
    - x represents total thickness of the absorber
  - Measuring beam intensity $I(x)$ in radiation detector
- As x increases, detector signal intensity decreases
  - From $I(x=0)$ measured with no absorber
  - To $I(x)$ measured with absorber of thickness $x > 0$



1.6. PHOTON INTERACTIONS WITH MATTER

1.6.1. Exponential absorption of photon beam in absorber

- An absorber of thickness *dx* reduces beam intensity by d*I(x)*
  - Fractional intensity reduction, –d*I(x)*/*I*(x) is proportional to:
    - Attenuation coefficient μ
    - Layer thickness d*x*

$$-\frac{\mathrm{d}I(x)}{I(x)} = \mu \mathrm{d}x$$

  - the negative sign indicates a decrease in signal *I*(*x*) with an increase in absorber thickness *x*

1.6. PHOTON INTERACTIONS WITH MATTER

1.6.1. Exponential absorption of photon beam in absorber

- ☐ Integrate over
  - absorber thickness *x* from 0 → *x*
  - over intensity *I*(*x*) from *I*(0) →*I*(*x*)

$$\int_{I(0)}^{I(x)} \frac{\mathrm{d}I(x)}{I(x)} = -\int_0^x \mu \mathrm{d}x$$

- ☐ Resulting in: $I(x) = I(0)\mathrm{e}^{-\mu x}$

  - Assuming μ is :
    - uniform in the absorber
    - independent of *x*

1.6. PHOTON INTERACTIONS WITH MATTER

1.6.2. Characteristic absorber thicknesses

☐ 3 special thicknesses used for characterization of photon beams:

- Half-value layer (HVL or $x_{1/2}$)
  - Absorber thickness that attenuates original $I(x)$ by 50 %

- Mean free path (MFP or $\bar{x}$)
  - Absorber thickness which attenuates beam intensity by 1/e = 36.8%

- Tenth-value layer (TVL or $x_{1/10}$)
  - Absorber thickness which attenuates beam intensity to 10% of original intensity

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1.6. PHOTON INTERACTIONS WITH MATTER

1.6.2. Characteristic absorber thicknesses

- HVL

$$I(x_{1/2}) = 0.5 I(0) = I(0)\mathrm{e}^{-\mu x_{1/2}}$$

$$\frac{1}{2} = \mathrm{e}^{-\mu x_{1/2}} \quad \text{or} \quad \mu x_{1/2} = \ln 2 = 0.693 \qquad \mathrm{HVL} = x_{1/2} = \frac{\ln 2}{\mu}$$

- MFP

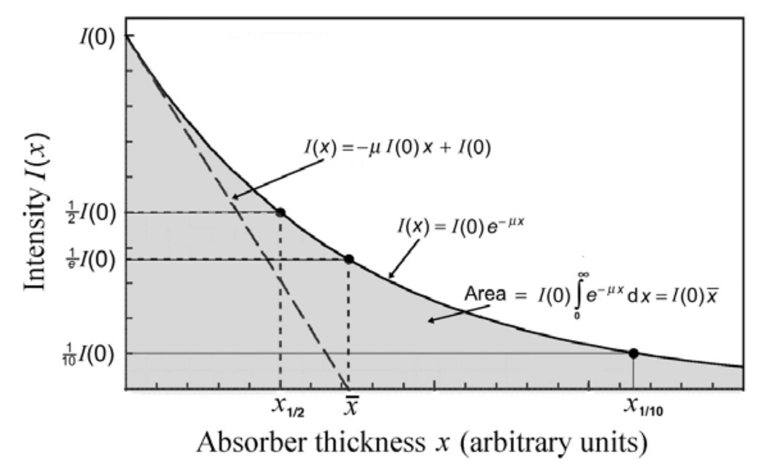
$$I(\bar{x}) = \frac{1}{\mathrm{e}} I(0) = 0.368 I(0) = I(0)\mathrm{e}^{-\mu \bar{x}}$$

$$\frac{1}{\mathrm{e}} = \mathrm{e}^{-\mu \bar{x}} \quad \text{or} \quad \mu \bar{x} = 1 \qquad \mathrm{MFP} = \bar{x} = \frac{1}{\mu}$$

- TVL

$$I(x_{1/10}) = 0.1 I(0) = I(0)\mathrm{e}^{-\mu x_{1/10}}$$

$$\frac{1}{10} = \mathrm{e}^{-\mu x_{1/10}} \quad \text{or} \quad \mu x_{1/10} = \ln 10 = 2.303 \qquad \mathrm{TVL} = x_{1/10} = \frac{\ln 10}{\mu}$$



$$I(x) = I(0)\mathrm{e}^{-\mu x}$$

1.6. PHOTON INTERACTIONS WITH MATTER

1.6.3. Attenuation coefficients

- ☐ In addition to the linear attenuation coefficient μ, other related attenuation coefficients and cross sections are used for describing photon beam attenuation:
  - Mass attenuation coefficient: μ_m
  - Atomic cross section: ${}_a\mu$
  - Electronic cross section: ${}_e\mu$

- ☐ The attenuation coefficients are related by:

$$\mu = \rho\mu_m = n^{\square}\ {}_a\mu = Zn^{\square}\ {}_e\mu \qquad\qquad n^{\square} = \frac{N_a}{V} = \rho\frac{N_a}{m} = \rho\frac{N_a}{A}$$

- □ absorber mass density
- $n^{\square}$ atoms N_a per volume V of absorber
- m absorber mass
- N_A Avogadro’s number
- $Zn^{\square}$ electrons per unit volume of absorber $\quad Zn^{\square} = \rho Z\frac{N_A}{A}$

1.6. PHOTON INTERACTIONS WITH MATTER

1.6.3. Attenuation coefficients

- ☐ Energy transfer coefficient $\mu_{\mathrm{tr}} = \mu \dfrac{\overline{E}_{\mathrm{tr}}}{h\nu}$
  - $\overline{E}_{\mathrm{tr}}$ = mean energy transferred from photons to charged particles (e^- and e^+) per unit path length.
  - $h\nu$ = primary photon energy
- ☐ Energy absorption coefficient $\mu_{\mathrm{ab}} = \mu \dfrac{\overline{E}_{\mathrm{ab}}}{h\nu}$
  - $\overline{E}_{\mathrm{ab}}$ = Mean energy absorbed in medium per unit path length
  - In the literature, μ_{en} is often used instead of μ_{ab}

1.6. PHOTON INTERACTIONS WITH MATTER

1.6.3. Attenuation coefficients

☐ Light charged particles (e^- & e^+) released/produced in absorbing medium through various photon interactions will either:

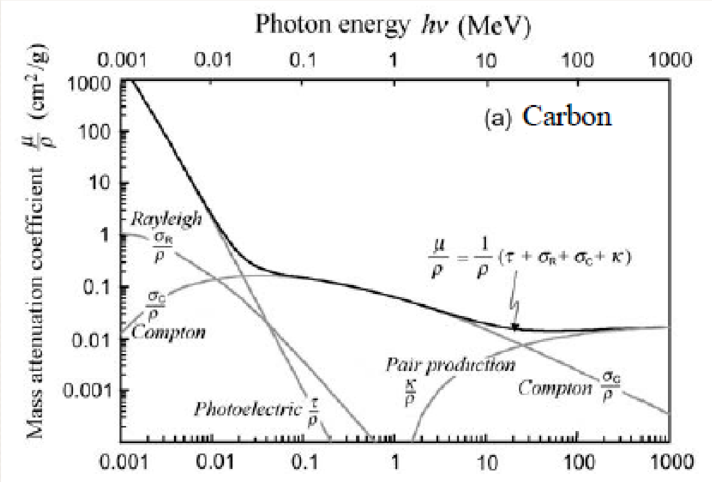
- Deposit energy to medium via Coulomb interactions w/ orbital electrons of absorbing medium (collision loss also referred to as ionization loss)
- Radiate E_K away as photons through Coulomb interactions with nuclei of absorbing medium (radiation loss)

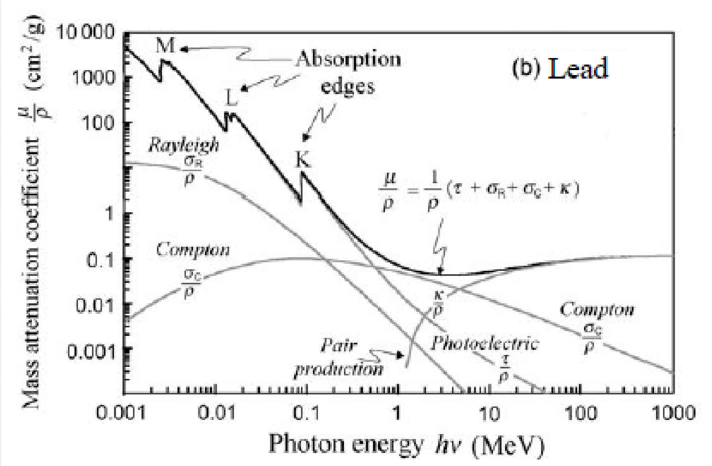
1.6. PHOTON INTERACTIONS WITH MATTER

1.6.3. Attenuation coefficients

- Typical examples mass attenuation coefficient μ/ρ plotted vs $h\nu$

- Observations for C (low *Z* absorber) & Pb (high *Z* absorber) for energy range: 0.001 - 1000 MeV
  - intermediate photon energies (~1 MeV)
    - Have similar μ/ρ □0.1 cm^2/g
  - For low photon energies
    - Pb μ/ρ >> C μ/ρ
  - at energies > 10 MeV
    - C μ/ρ essentially flat
    - Pb μ/ρ of lead increases with energy





1.6.4. Photon interactions on the microscopic scale

☐ Photons may experience various interactions with absorber atoms involving either of the following:

- Absorber nuclei
  - Photonuclear reaction: direct photon - nucleus interactions
  - Nuclear pair production: photon - electrostatic field of the nucleus interactions
- Orbital electrons of absorbing medium:
  - Compton effect, triplet production: photon - loosely bound electron interactions
  - Photoelectric effect, Rayleigh scattering: photon - tightly bound electron interactions

1.6. PHOTON INTERACTIONS WITH MATTER

1.6.4. Photon interactions on the microscopic scale

- ☐ Loosely bound electron
  - Binding energy $E_B << E_\gamma = h\nu$
  - Interactions considered to be between photon and ‘free’ (i.e. unbound) electron

- ☐ Tightly bound electron
  - E_B comparable to, larger than or slightly smaller than $E_\gamma = h\nu$
  - Interactions occur if E_B must be of the order of, but slightly smaller than $E_\gamma = h\nu$
    - i.e. $E_B \leq h\nu$
  - Interactions considered to be between photon and atom as a whole

1.6. PHOTON INTERACTIONS WITH MATTER

1.6.4. Photon interactions on the microscopic scale

- ☐ Two possible outcomes for photon after interaction with atom
  - Photon disappears and is absorbed completely
    - Photoelectric effect
    - Nuclear pair production
    - Triplet production
    - Photonuclear reaction
  - Photon scattered and changes direction but keeps its energy (Rayleigh scattering) or loses part of its energy (Compton effect)

1.6. PHOTON INTERACTIONS WITH MATTER

1.6.4. Photon interactions on the microscopic scale

- ☐ The most important photon interactions with atoms of the absorber are
  - Those with energetic electrons released from absorber atoms (and electronic vacancies left):
    - Compton effect
    - Photoelectric effect
    - Electronic pair production (triplet production)
  - Those with portion of the incident photon energy used to produce free electrons and positrons
    - Nuclear pair production
    - Photonuclear reactions

- ☐ All these light charged particles move through the absorber and either
  - Deposit E_K in the absorber (dose)
  - Transform part E_K into radiation bremsstrahlung radiation

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1.6. PHOTON INTERACTIONS WITH MATTER

1.6.4. Photon interactions on the microscopic scale

☐ Electronic vacancies from photon interactions with absorber atoms

- e^- from higher shell fills lower shell vacancy
- Transition energy emitted as one of the following:
  - Characteristic X ray (also called fluorescence photon)
  - Auger electron
  - This process continues until the vacancy migrates to the outer shell of the absorber atom
  - Free e^- from environment eventually fills outer shell vacancy
  - Absorber ion reverts to neutral atom in ground state

1.6.4. Photon interactions on the microscopic scale

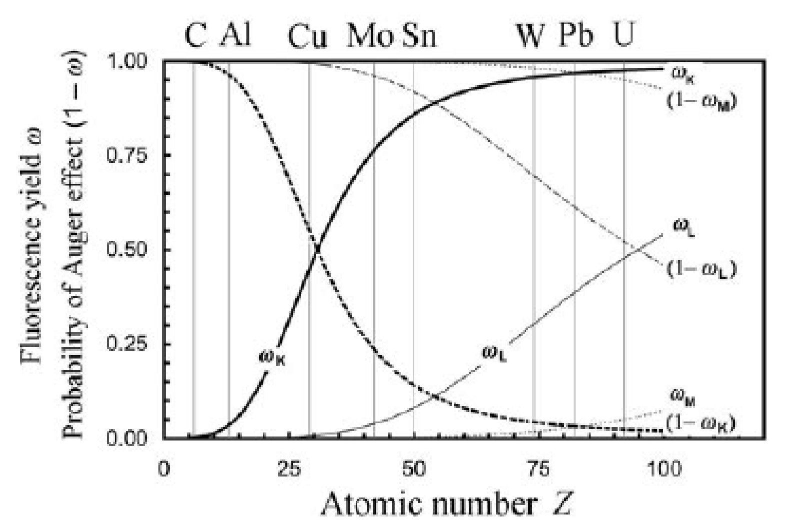
☐ Auger effect: Auger e^- emissions from excited atom

- Each Auger transition converts 1 vacancy into 2 vacancies
- Leads to cascade of low energy Auger e^-'s emitted from atom
- Auger e^-'s have very short range in tissue
- May produce ionization densities comparable to those in an alpha track
  - Biologically damaging

1.6. PHOTON INTERACTIONS WITH MATTER

1.6.4. Photon interactions on the microscopic scale

- Branching between characteristic γ and Auger e^- governed by fluorescence yield ω
  - $\omega =$ number of fluorescence γ's emitted per vacancy in given shell
  - ω also defined as probability of emission of fluorescence photon for a given shell vacancy
  - $(1 - \omega)$ gives probability of emission of Auger e^- for given shell vacancy



1.6. PHOTON INTERACTIONS WITH MATTER

1.6.5. Photoelectric effect

☐ Photoelectric effect:

- Only happens if photon energy $E_\gamma = h\nu > E_\mathrm{B}$
- Higher probability of happening when $h\nu$ is closer to E_B
- γ interacts with tightly bound electron, i.e. with whole atom
- Photon disappears
- Orbital electron ejected from atom as a photoelectron
- Ejected electron has kinetic energy E_K

$$E_\mathrm{K} = h\nu - E_\mathrm{B}$$

  - $h\nu$ = incident photon energy
  - E_B = binding energy of photoelectron

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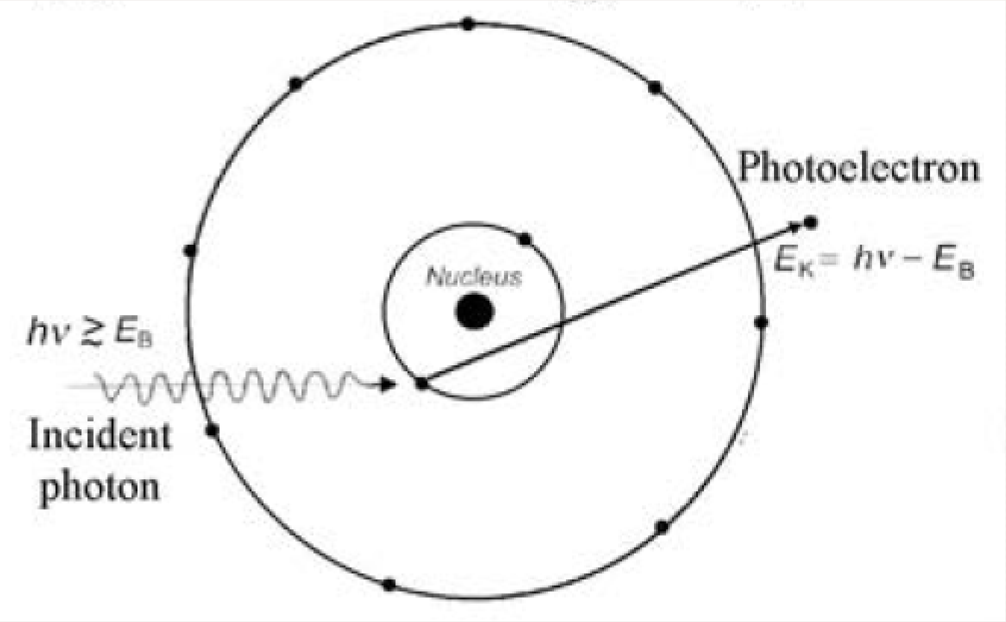
1.6. PHOTON INTERACTIONS WITH MATTER

1.6.5. Photoelectric effect

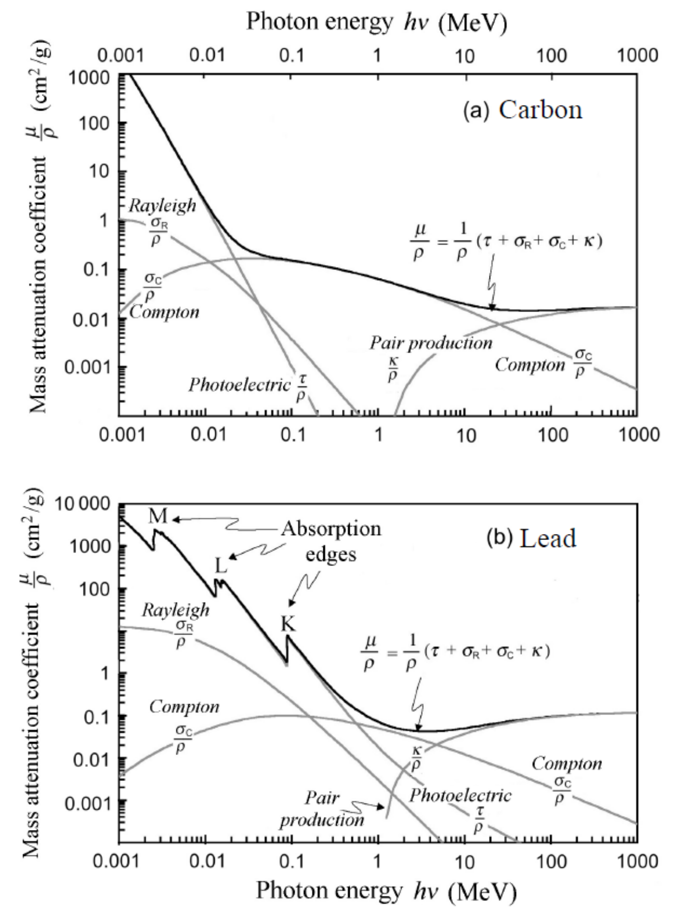
- Schematic diagram of the photoelectric effect
  - A photon interacts with an orbital electron
  - Electron is emitted from the atom as a photoelectron



1.6. PHOTON INTERACTIONS WITH MATTER

1.6.5. Photoelectric effect

- Photoelectric mass attenuation coefficient τ/ρ plotted for C & Pb (component of total attenuation coefficient μ/ρ)

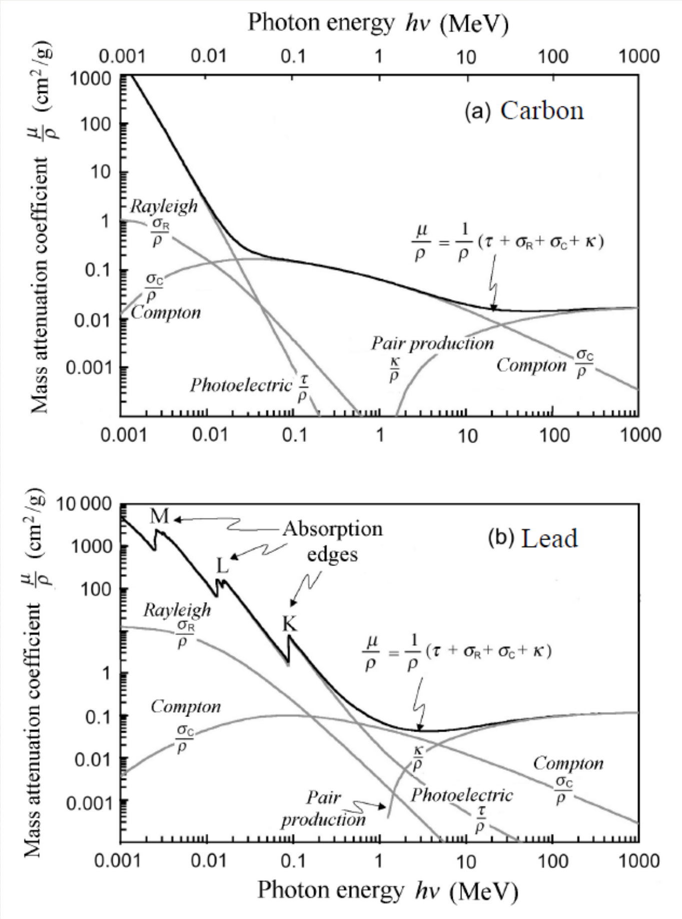


  - Absorption edges:
    - Sharp discontinuities when $h\nu = E_B$ of a given shell
    - e.g., K absorption edge
      - For Pb: E_B = 88 keV

1.6. PHOTON INTERACTIONS WITH MATTER

1.6.5. Photoelectric effect

- Photoelectric atomic attenuation coefficients
  - Atomic: ${}_{a}\tau \sim Z^{5}/(h\nu)^{3}$
  - Mass: $\tau_m = \tau/\rho \sim Z^{4}/(h\nu)^{3}$
- Photoelectric effect is the major contributor to μ/ρ at
  - Relatively low $E_\gamma = h\nu \sim E_B$ for K-shell
  - $E_\gamma < 0.1$ MeV
- At higher energies, major contributors to μ/ρ are
  - Compton effect ($E_\gamma \sim 1$MeV)
  - Pair production ($E_\gamma > 10$MeV)

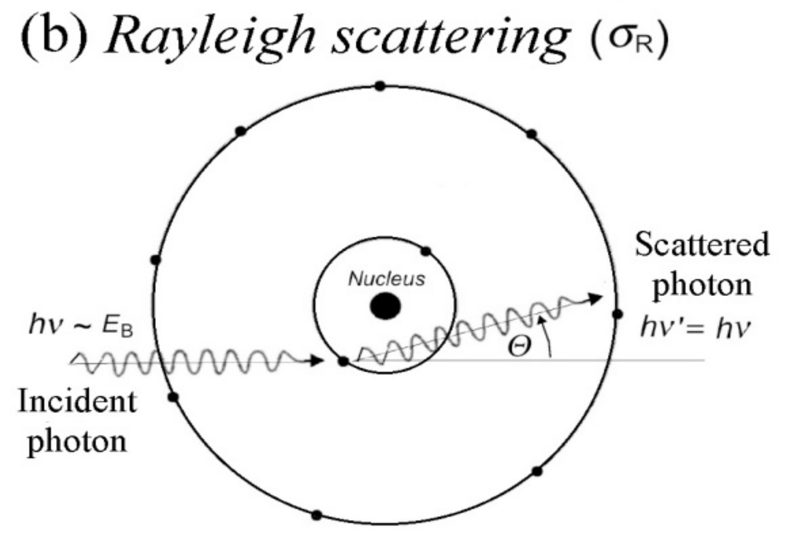


1.6. PHOTON INTERACTIONS WITH MATTER

1.6.6. Rayleigh (coherent) scattering

- ☐ Rayleigh (coherent) scattering
  - In coherent (Rayleigh) scattering the photon interacts with the full compliment of tightly bound atomic orbital electrons of the absorber atom
  - Elastic
    - Photon loses essentially none of its energy $h\nu$
    - Photon scattered through only a small angle θ

contributes to the attenuation coefficient



1.6. PHOTON INTERACTIONS WITH MATTER

1.6.6. Rayleigh (coherent) scattering

- ☐ Rayleigh (coherent) scattering
  - Contributes μ/ρ through elastic scattering process
  - Rayleigh atomic attenuation coefficient
    - ${}_a\sigma_R \sim Z^2/(h\nu)^2$
  - Rayleigh mass attenuation coefficient
    - $\sigma_R/\rho \sim Z/(h\nu)^2$

- ☐ Not important in radiation dosimetry because there’s no energy transfer from photons to charged particles in the absorber

- ☐ Amounts to only a few per cent of the total μ/ρ, but should not be neglected in attenuation calculations

1.6. PHOTON INTERACTIONS WITH MATTER

1.6.7. Compton effect ("incoherent scattering")

☐ Compton effect ("incoherent scattering" or "Compton scattering")

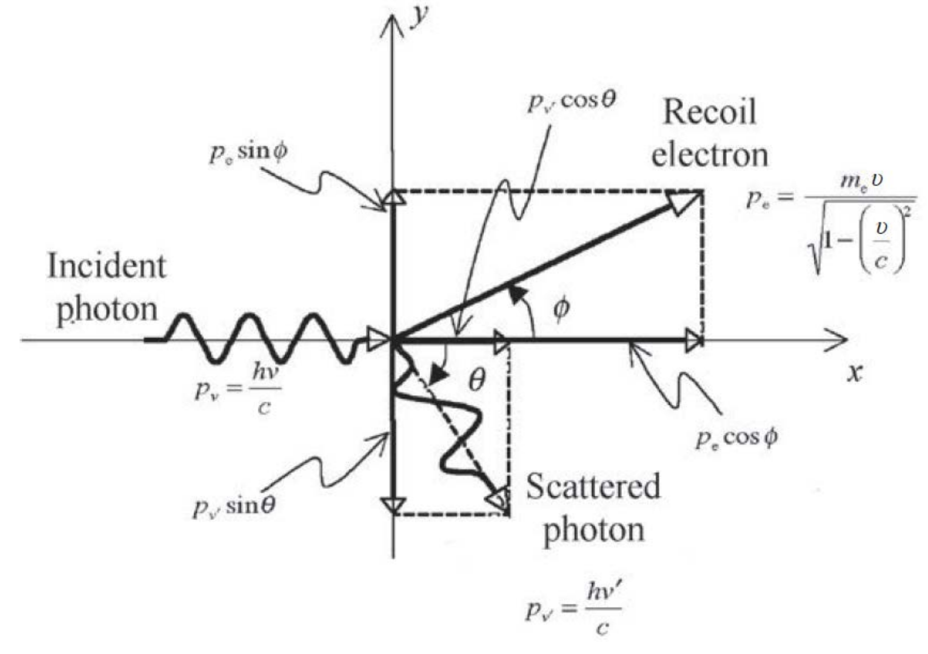
- Interaction between γ with $E_\gamma = h\nu$ and a loosely bound (“free”) e^-
  - ‘free’ because $E_\gamma \gg E_B$, i.e. loosely bound means essentially ‘free & stationary’

☐ Part of incident $E_\gamma = h\nu$ transferred to “free” orbital electron which is emitted from the atom as the Compton (recoil) electron

1.6. PHOTON INTERACTIONS WITH MATTER

1.6.7. Compton effect (incoherent scattering)

- Photon is scattered through scattering angle θ & its energy $E'_\gamma = h\nu'$ is lower than $E_\gamma = h\nu$ (incident photon energy)
- Angle ϕ represents the angle between the incident γ direction and the Compton e^- direction



1.6. PHOTON INTERACTIONS WITH MATTER

1.6.7. Compton effect (incoherent scattering)

- Conservation of energy

$$h\nu + m_e c^2 = h\nu' + m_e c^2 + E_K$$

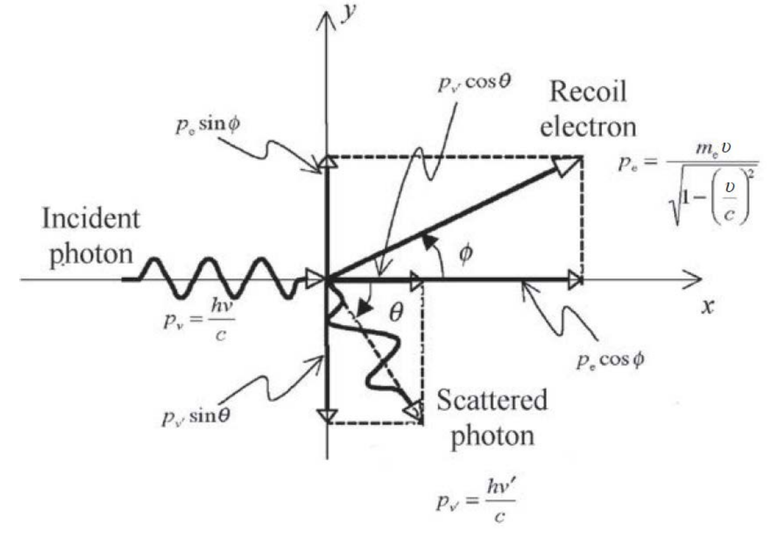
$$h\nu = h\nu' + E_K$$

- Conservation of momentum (x axis)

$$p_\nu = \frac{h\nu'}{c}\cos\theta + \frac{m_e\upsilon}{\sqrt{1-\frac{\upsilon^2}{c^2}}}\cos\phi$$

- Conservation of momentum (y axis)

$$0 = -\frac{h\nu'}{c}\sin\theta + \frac{m_e\upsilon}{\sqrt{1-\frac{\upsilon^2}{c^2}}}\sin\phi$$



where

$m_e c^2$ rest energy of electron (0.511 MeV)

E_K kinetic energy of recoil (Compton) electron

υ velocity of recoil (Compton) electron

c speed of light in a vacuum (3×10^8 m/s)

1.6. PHOTON INTERACTIONS WITH MATTER

1.6.7. Compton effect (incoherent scattering)

- Basic Compton equation (also referred to as the Compton wavelength-shift equation) follows from conservation of energy & momentum:

$$\lambda' - \lambda = \Delta\lambda = \frac{h}{m_e c}(1 - \cos\theta) = \lambda_C(1 - \cos\theta)$$

λ = wavelength of the incident photon (c/ν)

λ' = wavelength of the scattered photon (c/ν')

$\Delta\lambda$ = wavelength shift in Compton effect ($\lambda' - \lambda$)

λ_C = Compton wavelength of the electron = 0.024Å

1.6. PHOTON INTERACTIONS WITH MATTER

1.6.7. Compton effect (incoherent scattering)

- ☐ Relationship between the scattered E_γ & incident E_γ is:

$$h\nu'(h\nu,\theta) = h\nu \frac{1}{1+\varepsilon(1-\cos\theta)} \qquad \varepsilon = \frac{h\nu}{m_\mathrm{e}c^2}$$

- ☐ Relationship between the E_K of recoil electron & incident E_γ is:

$$E_\mathrm{K}^\mathrm{C}(h\nu,\theta) = h\nu - h\nu' = h\nu - h\nu\frac{1}{1+\varepsilon(1-\cos\theta)} = h\nu\frac{\varepsilon(1+\cos\theta)}{1+\varepsilon(1-\cos\theta)} \qquad \varepsilon = \frac{h\nu}{m_\mathrm{e}c^2}$$

- ☐ Scattering θ & recoil ϕ angles are related as:

$$\cot\phi = (1+\varepsilon)\tan\frac{\theta}{2} \qquad \varepsilon = \frac{h\nu}{m_\mathrm{e}c^2}$$

1.6. PHOTON INTERACTIONS WITH MATTER

1.6.7. Compton effect (incoherent scattering)

- Energy of:
  - forward scattered photons ($\theta = 0$) $h\nu'|_{\theta=0} = h\nu$
  - side-scattered photons ($\theta = \pi/2$) $h\nu'|_{\theta=\frac{\pi}{2}} = \dfrac{h\nu}{1+\varepsilon}$
  - back-scattered photons ($\theta = \pi$) $h\nu'|_{\theta=\pi} = \dfrac{h\nu}{1+2\varepsilon}$
- For $h\nu \to \infty$
  - $\theta = 0$ $h\nu'|_{\theta=0} = h\nu$
  - $\theta = \pi/2$ $h\nu'|_{\theta=\frac{\pi}{2}} = m_e c^2$
  - $\theta = \pi$ $h\nu'|_{\theta=\pi} = \dfrac{m_e c^2}{2}$

1.6. PHOTON INTERACTIONS WITH MATTER

1.6.7. Compton effect (incoherent scattering)

- ${}_{e}\sigma_{C}$ (Compton electronic attenuation coefficient)
  - Steadily decreases with increasing $h\nu$
    - Theoretical value = 0.665×10^{-24} cm^2/electron (Thomson cross-section) at low E_γ
    - 0.21×10^{-24} cm^2/electron at $h\nu = 1$ MeV
    - 0.51×10^{-24} cm^2/electron at $h\nu = 10$ MeV
    - 0.008×10^{-24} cm^2/electron at $h\nu = 100$ MeV
  - Independent of Z
    - For C($Z = 6$) and Pb($Z = 82$) at E_γ ~1 MeV, where Compton effect predominates, both are □0.1 cm^2/electron irrespective of Z
- ${}_{a}\sigma_{C}$ (Compton atomic attenuation coefficient)
  - Depends linearly on absorber Z (because Compton interaction is with free electron)

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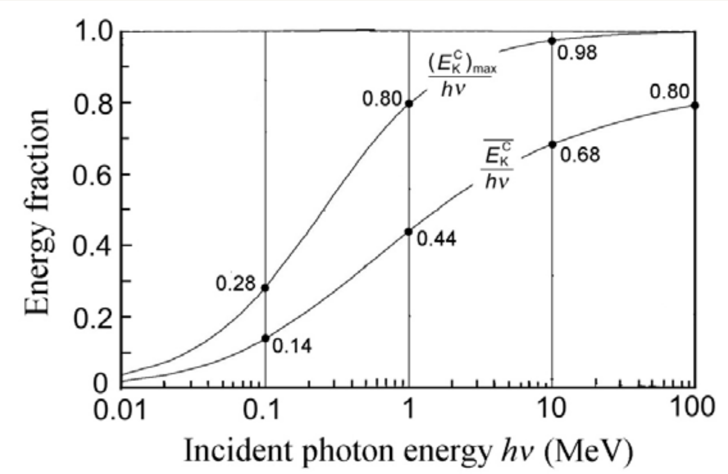
1.6. PHOTON INTERACTIONS WITH MATTER

1.6.7. Compton effect (incoherent scattering)

- Compton maximum energy transfer fraction $(f_C)_{max}$:
  - Maximum energy transfer to recoil electron occurs when photon is back-scattered ($\theta = \pi$)

$$(f_C)_{max} = \frac{(E_K^C)_{max}}{h\nu} = \frac{2\varepsilon}{1+2\varepsilon}$$

- Mean energy transferred to the Compton electron normalized by $h\nu$
  - Very important in radiation dosimetry
  - fractional energy, $\bar{f}_C$, transfer to recoil electrons is
    - $\bar{f}_C = 0.02$ at $h\nu = 0.01$ MeV
    - Rises and then reaches 1 asymptotically at very high $h\nu$



1.6. PHOTON INTERACTIONS WITH MATTER

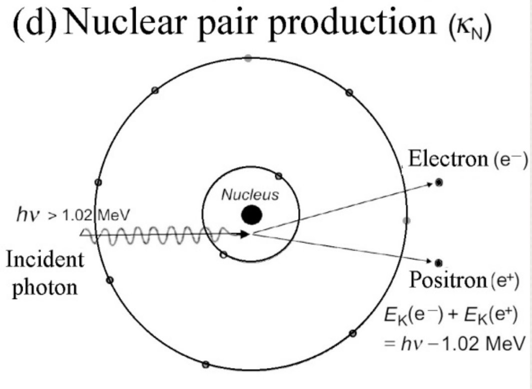
1.6.8. Pair production

- ☐ Pair production
  - Production of e^- - e^+ pair + complete absorption of incident photon by absorber atom
  - Happens if : $E_\gamma = h\nu > 2m_ec^2 = 1.022$ MeV, with m_ec^2 = rest energy of e^- & e^+
- ☐ Conserves:
  - Energy
  - Charge
  - Momentum

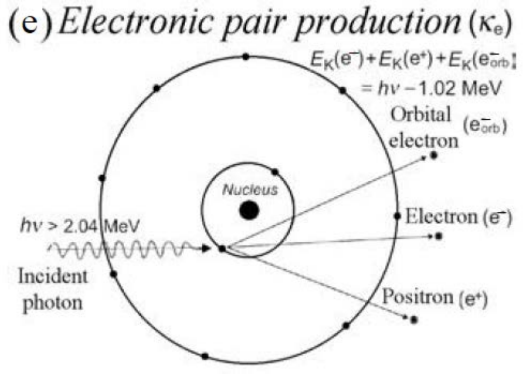
1.6. PHOTON INTERACTIONS WITH MATTER

1.6.8. Pair production

- Two types of pair production are known:
  - Nuclear pair production
    - Collision partner is absorber atomic nucleus
    - Characterized by: $E_\gamma > 2m_e c^2 = 1.022$ MeV



  - Electronic pair production or triplet production
    - Less probable
    - Pair production in Coulomb field of absorber orbital electron
    - Threshold: $E_\gamma > 4m_e c^2 = 2.044$ MeV



1.6. PHOTON INTERACTIONS WITH MATTER

1.6.8. Pair production

- ☐ Pair production attenuation coefficients
  - Usually as one parameter for nuclear & electronic
  - Nuclear pair production contributes > 90%
  - Pair production atomic attenuation coefficient ${}_{a}\kappa$
    - ${}_{a}\kappa \sim Z^2$
  - Pair production mass attenuation coefficient κ/ρ
    - $\kappa/\rho \sim Z$

- ☐ Pair production probability
  - Zero for $E_\gamma < 2m_ec^2 = 1.022\ MeV$
  - Increases rapidly with E_γ > threshold

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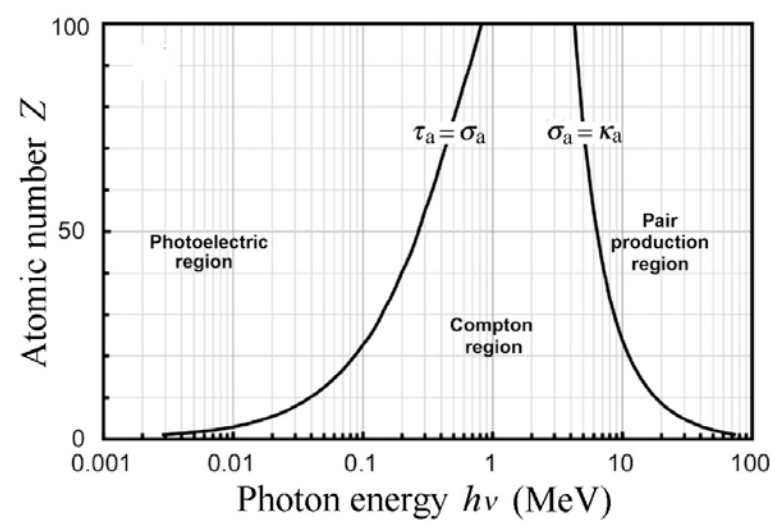
1.6. PHOTON INTERACTIONS WITH MATTER

1.6.9. Relative predominance of individual effects

- The probability for a photon to undergo any one of the various interactions absorber depends on:
  - Photon energy $h\nu$
  - Absorber Z
  - Pair production at high E_γ
  - Photoelectric effect generally predominates at low E_γ
  - Compton effect generally predominates at intermediate E_γ



1.6. PHOTON INTERACTIONS WITH MATTER

1.6.10. Macroscopic attenuation coefficients

- For a given $h\nu$ & Z:
  - Linear attenuation coefficient μ
  - Linear energy transfer coefficient μ_{tr}
  - Linear energy absorption coefficient μ_{ab} (often designated μ_{en})

are given as a sum of coefficients for individual photon interactions

$$\mu = \rho \frac{N_{\mathrm{A}}}{A}\left({}_{\mathrm{a}}\tau + {}_{\mathrm{a}}\sigma_{\mathrm{R}} + {}_{\mathrm{a}}\sigma_{\mathrm{C}} + {}_{\mathrm{a}}\kappa\right)$$

$$\mu_{\mathrm{tr}} = \rho \frac{N_{\mathrm{A}}}{A}\left[{}_{\mathrm{a}}\tau_{\mathrm{tr}} + \left({}_{\mathrm{a}}\sigma_{\mathrm{C}}\right)_{\mathrm{tr}} + {}_{\mathrm{a}}\kappa_{\mathrm{tr}}\right] = \rho \frac{N_{\mathrm{A}}}{A}\left[{}_{\mathrm{a}}\tau\,\overline{f}_{\mathrm{PE}} + {}_{\mathrm{a}}\sigma_{\mathrm{C}}\,\overline{f}_{\mathrm{C}} + {}_{\mathrm{a}}\kappa\,\overline{f}_{\mathrm{PP}}\right]$$

$$\mu_{\mathrm{ab}} = \mu_{\mathrm{en}} = \mu_{\mathrm{tr}}\left(1 - \overline{g}\right)$$

$\overline{g}$ = fraction of mean energy transferred from photons to charged articles subsequently lost by charged articles through radiation losses