CHAPTER 1

BASIC PHYSICS FOR NUCLEAR MEDICINE

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1.1. INTRODUCTION

The technologies used in nuclear medicine for diagnostic imaging have evolved over the last century, starting with Röntgen's discovery of X rays and Becquerel's discovery of natural radioactivity. Each decade has brought innovation in the form of new equipment, techniques, radiopharmaceuticals, advances in radionuclide production and, ultimately, better patient care. All such technologies have been developed and can only be practised safely with a clear understanding of the behaviour and principles of radiation sources and radiation detection. These central concepts of basic radiation physics and nuclear physics are described in this chapter and should provide the requisite knowledge for a more in depth understanding of the modern nuclear medicine technology discussed in subsequent chapters.

1.1.1. Fundamental physical constants

The chapter begins with a short list of physical constants of importance to general physics as well as to nuclear and radiation physics. The data listed below were taken from the CODATA set of values issued in 2006 and are available from a web site supported by the National Institute of Science and Technology in Washington, DC, United States of America: http://physics.nist.gov/cuu/Constants

- Avogadro's number: $N_A = 6.022 \times 10^{23}$ mol^{-1} or 6.022×10^{23} atoms/mol.
- Speed of light in vacuum: $c = 2.998 \times 10^8$ m/s $\approx 3 \times 10^8$ m/s.
- Electron charge: $e = 1.602 \times 10^{-19}$ C.
- Electron and positron rest mass: $m_e = 0.511$ MeV/c^2.
- Proton rest mass: $m_p = 938.3$ MeV/c^2.
- Neutron rest mass: $m_n = 939.6$ MeV/c^2.
- Atomic mass unit: u = 931.5 MeV/c^2.
- Planck's constant: $h = 6.626 \times 10^{-34}$ J · s.
- Electric constant (permittivity of vacuum): $\varepsilon_0 = 8.854 \times 10^{-12}$ C · V^{-1} · m^{-1}.
- Magnetic constant (permeability of vacuum): $\mu_0 = 4\pi \times 10^{-7}$ V · s · A^{-1} · m^{-1}.
- Newtonian gravitation constant: $G = 6.672 \times 10^{-11}$ m^3 · kg^{-1} · s^{-2}.
- Proton mass/electron mass: $m_p/m_e = 1836.0$.
- Specific charge of electron: $e/m_e = 1.758 \times 10^{11}$ C/kg.

1.1.2. Physical quantities and units

A physical quantity is defined as a quantity that can be used in mathematical equations of science and technology. It is characterized by its numerical value (magnitude) and associated unit. The following rules apply to physical quantities and their units in general:

- Symbols for physical quantities are set in italics (sloping type), while symbols for units are set in roman (upright) type (e.g. m = 21 kg; E = 15 MeV; K = 220 Gy).
- Superscripts and subscripts used with physical quantities are set in italics if they represent variables, quantities or running numbers; they are in roman type if they are descriptive (e.g. N_x, λ_m but λ_{max}, E_{ab}, μ_{tr}).
- Symbols for vector quantities are set in bold italics.

The currently used metric system of units is known as the International System of Units (SI). The system is founded on base units for seven basic physical quantities. All other quantities and units are derived from the seven base quantities and units. The seven base SI quantities and their units are:

(a) Length l: metre (m).
(b) Mass m: kilogram (kg).
(c) Time t: second (s).
(d) Electric current I: ampere (A).

(e) Temperature T: kelvin (K).
(f) Amount of substance: mole (mol).
(g) Luminous intensity: candela (cd).

Examples of basic and derived physical quantities and their units are given in Table 1.1.

TABLE 1.1. BASIC QUANTITIES AND SEVERAL DERIVED PHYSICAL QUANTITIES AND THEIR UNITS IN THE INTERNATIONAL SYSTEM OF UNITS AND IN RADIATION PHYSICS

Physical quantity	Symbol	SI unit	Units commonly used in radiation physics	Conversion
Length	l	m	nm, Å, fm	1 m = 10^9 nm = 10^{10} Å = 10^{15} fm
Mass	m	kg	MeV/c^2	1 MeV/c^2 = 1.78×10^{-30} kg
Time	t	s	ms, μs, ns, ps	1 s = 10^3 ms = 10^6 μs = 10^9 ns = 10^{12} ps
Current	I	A	mA, μA, nA, pA	1 A = 10^3 mA = 10^6 μA = 10^9 nA
Temperature	T	K		T (in K) = T (in °C) + 273.16
Mass density	ρ	kg/m^3	g/cm^3	1 kg/m^3 = 10^{-3} g/cm^3
Current density	j	A/m^2		
Velocity	υ	m/s		
Acceleration	a	m/s^2		
Frequency	ν	Hz		1 Hz = 1 s^{-1}
Electric charge	q	C	e	1 e = 1.602×10^{-19} C
Force	F	N		1 N = 1 kg · m · s^{-2}
Pressure	P	Pa	760 torr = 101.3 kPa	1 Pa = 1 N/m^2 = 7.5×10^{-3} torr
Momentum	p	N · s		1 N · s = 1 kg · m · s^{-1}
Energy	E	J	eV, keV, MeV	1 eV = 1.602×10^{-19} J = 10^{-3} keV
Power	P	W		1 W = 1 J/s = 1 V · A

1.1.3. Classification of radiation

Radiation, the transport of energy by electromagnetic waves or atomic particles, can be classified into two main categories depending on its ability to ionize matter. The ionization potential of atoms, i.e. the minimum energy required to ionize an atom, ranges from a few electronvolts for alkali elements to 24.6 eV for helium which is in the group of noble gases. Ionization potentials for all other atoms are between the two extremes.

- Non-ionizing radiation cannot ionize matter because its energy per quantum is below the ionization potential of atoms. Near ultraviolet radiation, visible light, infrared photons, microwaves and radio waves are examples of non-ionizing radiation.
- Ionizing radiation can ionize matter either directly or indirectly because its quantum energy exceeds the ionization potential of atoms. X rays, γ rays, energetic neutrons, electrons, protons and heavier particles are examples of ionizing radiation.

1.1.4. Classification of ionizing radiation

Ionizing radiation is radiation that carries enough energy per quantum to remove an electron from an atom or a molecule, thus introducing a reactive and potentially damaging ion into the environment of the irradiated medium. Ionizing radiation can be categorized into two types: (i) directly ionizing radiation and (ii) indirectly ionizing radiation. Both directly and indirectly ionizing radiation can traverse human tissue, thereby enabling the use of ionizing radiation in medicine for both imaging and therapeutic procedures.

- Directly ionizing radiation consists of charged particles, such as electrons, protons, α particles and heavy ions. It deposits energy in the medium through direct Coulomb interactions between the charged particle and orbital electrons of atoms in the absorber.
- Indirectly ionizing radiation consists of uncharged (neutral) particles which deposit energy in the absorber through a two-step process. In the first step, the neutral particle releases or produces a charged particle in the absorber which, in the second step, deposits at least part of its kinetic energy in the absorber through Coulomb interactions with orbital electrons of the absorber in the manner discussed above for directly ionizing charged particles.

1.1.5. Classification of indirectly ionizing photon radiation

Indirectly ionizing photon radiation consists of three main categories: (i) ultraviolet, (ii) X ray and (iii) γ ray. Ultraviolet photons are of limited use in medicine. Radiation used in imaging and/or treatment of disease consists mostly of photons of higher energy, such as X rays and γ rays. The commonly accepted difference between the two is based on the radiation's origin. The term 'γ ray' is reserved for photon radiation that is emitted by the nucleus or from other particle decays. The term 'X ray', on the other hand, refers to radiation emitted by electrons, either orbital electrons or accelerated electrons (e.g. bremsstrahlung type radiation).

With regard to their origin, the photons of the indirectly ionizing radiation type fall into four categories: characteristic (fluorescence) X rays, bremsstrahlung X rays, photons resulting from nuclear transitions and annihilation quanta.

1.1.6. Characteristic X rays

Orbital electrons have a natural tendency to configure themselves in such a manner that they inhabit a minimal energy state for the atom. When a vacancy is opened within an inner shell, as a result of an ionization or excitation process, an outer shell electron will make a transition to fill the vacancy, usually within a nanosecond for solid materials. The energy liberated in this transition may be released in the form of a characteristic (fluorescence) photon of energy equal to the difference between the binding energies of the initial and final vacancies. Since different elements have different binding energies for their electronic shells, the energy of the photon released in this process will be characteristic of the particular atom. Rather than being emitted as a characteristic photon, the transition energy may also be transferred to an orbital electron that is then emitted with kinetic energy that is equal to the transition energy less the electron binding energy. The emitted orbital electron is called an Auger electron.

1.1.7. Bremsstrahlung

The word 'bremsstrahlung' can be translated from its original German term as 'braking radiation', and is a name aptly assigned to the phenomenon. When light charged particles (electrons and positrons) are slowed down or 'negatively' accelerated (decelerated) by interactions with other charged particles in matter (e.g. by atomic nuclei), the kinetic energy that they lose is converted to electromagnetic radiation, referred to as bremsstrahlung radiation. The energy spectrum of bremsstrahlung is non-discrete (i.e. continuous) and ranges between zero and the kinetic energy of the initial charged particle. Bremsstrahlung plays a central role in modern imaging and therapeutic equipment, since it can be used to produce X rays on demand from an electrical energy source. The power emitted in the form of bremsstrahlung photons is proportional to the square of the particle's charge and the square of the particle's acceleration.

1.1.8. Gamma rays

When a nuclear reaction or spontaneous nuclear decay occurs, the process may leave the product (daughter) nucleus in an excited state. The nucleus can then make a transition to a more stable state by emitting a γ ray photon and the process is referred to as γ decay. The energy of the photon emitted in γ decay is characteristic of the nuclear energy transition, but the recoil of the emitting atom produces a spectrum centred on the characteristic energy. Gamma rays typically have energies above 100 keV and wavelengths less than 0.1 Å.

1.1.9. Annihilation quanta

When a parent nucleus undergoes β plus decay or a high energy photon interacts with the electric field of either the nucleus or the orbital electron, an energetic positron may be produced. In moving through an absorber medium, the positron loses most of its kinetic energy as a result of Coulomb interactions with absorber atoms. These interactions result in collision loss when the interaction is with an orbital electron of the absorber atom and in radiation loss (bremsstrahlung) when the interaction is with the nucleus of the absorber atom. Generally, after the positron loses all of its kinetic energy through collision and radiation losses, it will undergo a final collision with an available orbital electron (due to the Coulomb attractive force between the positively charged positron and a local negatively charged electron) in a process called positron annihilation. During annihilation, the positron and electron disappear and are replaced by two oppositely directed annihilation quanta, each with an energy of 0.511 MeV. This process satisfies a number of conservation laws: conservation of electric charge, conservation of linear momentum, conservation of angular momentum and conservation of total energy.

A percentage of positron annihilations occur before the positron expends all of its kinetic energy and the process is then referred to as in-flight annihilation. The two quanta emitted in in-flight annihilation are not of identical energies and do not necessarily move in absolute opposite directions.

1.1.10. Radiation quantities and units

Accurate measurement of radiation is very important in all medical uses of radiation, be it for diagnosis or treatment of disease. In diagnostic imaging procedures, image quality must be optimized, so as to obtain the best possible image with the lowest possible radiation dose to the patient to minimize the risk of morbidity. In radiotherapy, the prescribed dose must be delivered accurately and precisely to maximize the tumour control probability (TCP) and to minimize the normal tissue complication probability (NTCP). In both instances, the risk of morbidity includes acute radiation effects (radiation injury) as well as late radiation-induced effects, such as induction of cancer and genetic damage.

Several quantities and units were introduced for the purpose of quantifying radiation and the most important of these are listed in Table 1.2. Also listed are the definitions for the various quantities and the relationships between the old units and the SI units for these quantities. The definitions of radiation related physical quantities are as follows:

- *Exposure X* is related to the ability of photons to ionize air. Its unit, roentgen (R), is defined as a charge of 2.58×10^{-4} coulombs produced per kilogram of air.
- *Kerma K* (acronym for kinetic energy released in matter) is defined for indirectly ionizing radiation (photons and neutrons) as energy transferred to charged particles per unit mass of the absorber.
- *Dose* (also referred to as absorbed dose) is defined as energy absorbed per unit mass of medium. Its SI unit, gray (Gy), is defined as 1 joule of energy absorbed per kilogram of medium.
- *Equivalent dose* H_T is defined as the dose multiplied by a radiation weighting factor w_R. When different types of radiation are present, H_T is defined as the sum of all of the individual weighted contributions. The SI unit of equivalent dose is the sievert (Sv).
- *Effective dose E* of radiation is defined as the equivalent dose H_T multiplied by a tissue weighting factor w_T. The SI unit of effective dose is also the sievert (Sv).
- *Activity* $\mathcal{A}$ of a radioactive substance is defined as the number of nuclear decays per time. Its SI unit, becquerel (Bq), corresponds to one decay per second.

TABLE 1.2. RADIATION QUANTITIES, UNITS AND CONVERSION BETWEEN OLD AND SI UNITS

Quantity	Definition	SI unit	Old unit	Conversion
Exposure X	$X = \frac{\Delta Q}{\Delta m_{\mathrm{air}}}$	$2.58 \times \frac{10^{-4}\,\mathrm{C}}{\mathrm{kg\ air}}$	$1\ \mathrm{R} = \frac{1\ \mathrm{esu}}{\mathrm{cm}^3\ \mathrm{air}_{\mathrm{STP}}}$	$1\ \mathrm{R} = 2.58 \times \frac{10^{-4}\mathrm{C}}{\mathrm{kg\ air}}$
Kerma K	$K = \frac{\Delta E_{\mathrm{tr}}}{\Delta m}$	$1\ \mathrm{Gy} = 1 \frac{\mathrm{J}}{\mathrm{kg}}$	—	—
Dose D	$D = \frac{\Delta E_{\mathrm{ab}}}{\Delta m}$	$1\ \mathrm{Gy} = 1 \frac{\mathrm{J}}{\mathrm{kg}}$	$1\ \mathrm{rad} = 100\ \frac{\mathrm{erg}}{\mathrm{g}}$	1 Gy = 100 rad
Equivalent dose H_T	$H_T = D w_R$	1 Sv	1 rem	1 Sv = 100 rem
Effective dose E	$E = H_T w_T$	1 Sv	1 rem	1 Sv = 100 rem
Activity $\mathcal{A}$	$\mathcal{A} = \lambda N$	$1\ \mathrm{Bq} = 1\ \mathrm{s}^{-1}$	$1\ \mathrm{Ci} = 3.7 \times 10^{10}\ \mathrm{s}^{-1}$	$1\ \mathrm{Bq} = \frac{1\ \mathrm{Ci}}{3.7 \times 10^{10}}$

1.2. BASIC DEFINITIONS FOR ATOMIC STRUCTURE

The constituent particles forming an atom are protons, neutrons and electrons. Protons and neutrons are known as nucleons and form the nucleus of the atom. Protons have a positive charge, neutrons are neutral and electrons have a negative charge mirroring that of a proton. In comparison to electrons, protons and neutrons have a relatively large mass exceeding the electron mass by a factor of almost 2000 (note: $m_p/m_e = 1836$).

The following general definitions apply to atomic structure:

- Atomic number Z is the number of protons and number of electrons in an atom.
- Atomic mass number A is the number of nucleons in an atom, i.e. the number of protons Z plus the number of neutrons N in an atom: $A = Z + N$.
- Atomic mass m_a is the mass of a specific isotope expressed in atomic mass units u, where 1 u is equal to one twelfth of the mass of the ^{12}C atom (unbound, at rest and in the ground state) or 931.5 MeV/c^2. The atomic mass is smaller than the sum of the individual masses of the constituent particles because of the intrinsic energy associated with binding the particles (nucleons) within the nucleus. On the other hand, the atomic mass is larger than the nuclear mass M because the atomic mass includes the mass contribution of Z orbital

electrons while the nuclear mass M does not. The binding energy of orbital electrons to the nucleus is ignored in the definition of the atomic mass.

While for ^{12}C the atomic mass is exactly 12 u, for all other atoms m_a does not exactly match the atomic mass number A. However, for all atomic entities, A (an integer) and m_a are very similar to one another and often the same symbol (A) is used for the designation of both. The mass in grams equal to the average atomic mass of a chemical element is referred to as the mole (mol) of the element and contains exactly 6.022×10^{23} atoms. This number is referred to as the Avogadro constant N_A of entities per mole. The atomic mass number of all elements is, thus, defined such that A grams of every element contain exactly N_A atoms. For example, the atomic mass of natural cobalt is 58.9332 u. Thus, one mole of natural cobalt has a mass of 58.9332 g and by definition contains 6.022×10^{23} entities (cobalt atoms) per mole of cobalt.

The number of atoms N_a per mass of an element is given as:

$$\frac{N_a}{m} = \frac{N_A}{A} \tag{1.1}$$

The number of electrons per volume of an element is:

$$Z\frac{N_a}{V} = \rho Z\frac{N_a}{m} = \rho Z\frac{N_A}{A} \tag{1.2}$$

The number of electrons per mass of an element is:

$$Z\frac{N_a}{m} = Z\frac{N_A}{A} \tag{1.3}$$

It should be noted that $Z/A \approx 0.5$ for all elements with one notable exception of hydrogen for which $Z/A = 1$. Actually, Z/A slowly decreases from 0.5 for low Z elements to 0.4 for high Z elements. For example, Z/A for ^{4}He is 0.5, for ^{60}Co is 0.45 and for ^{235}U is 0.39.

If it is assumed that the mass of a molecule is equal to the sum of the masses of the atoms that make up the molecule, then, for any molecular compound, there are N_A molecules per mole of the compound where the mole in grams is defined as the sum of the atomic mass numbers of the atoms making up the molecule. For example, 1 mole of water (H_2O) is 18 g of water and 1 mole of carbon dioxide (CO_2) is 44 g of carbon dioxide. Thus, 18 g of water or 44 g of carbon dioxide contain exactly N_A molecules (or 3 N_A atoms, since each molecule of water and carbon dioxide contains three atoms).

1.2.1. Rutherford model of the atom

At the beginning of the 20th century, the structure of the atom was not well known. Scientific pioneers such as Dalton, Mendeleev and Thomson, among others, were developing a common theory through their endeavours. Often noted as a significant contribution to the modern understanding of the atom, is the work performed by Rutherford and his colleagues Geiger and Marsden in 1909. Through observation of the behaviour of positively charged α particles traversing a thin gold foil, Rutherford concluded that the positive charge and most of the mass of the atom are concentrated in the atomic nucleus (diameter of a few femtometres) and negative electrons are spread over the periphery of the atom (diameter of a few ångströms). This work was significant because it introduced a new specialty of physics (nuclear physics) and demonstrated that the atom is not simply a single particle, but instead is made up of smaller subatomic particles, organized in an atom with well defined characteristics.

1.2.2. Bohr model of the hydrogen atom

Bohr expanded the Rutherford atomic model in 1913 using a set of four postulates that combine classical, non-relativistic mechanics with the concept of angular momentum quantization. The Bohr model of the atom can be said to resemble a ‘planetary model’ in that the protons and neutrons occupy a dense central region called the nucleus and the electrons orbit the nucleus as planets orbit the sun. The Bohr model introduces the concept that the angular momenta of orbital electrons revolving around the nucleus in allowed orbits, radii of the allowed electronic orbits (shells), velocities of orbital electrons in allowed orbits and binding energies of orbital electrons in allowed orbits within the atom, are restricted to certain discrete states. This means that angular momenta, radii, velocities and binding energies of orbital electrons are quantized.

While scientific theory was later expanded to include the necessary principles of quantum mechanics in our understanding of the atom, the Bohr model is elegant and provides a simplistic, yet practical, view of the atom that is still used for teaching atomic principles, and successfully deals with one-electron entities, such as the hydrogen atom, the singly ionized helium atom and the doubly ionized lithium atom.

1.3. BASIC DEFINITIONS FOR NUCLEAR STRUCTURE

According to the Rutherford–Bohr atomic model, most of the atomic mass is concentrated in the atomic nucleus consisting of Z protons and $(A - Z)$ neutrons, where Z is the atomic number and A the atomic mass number of a given nucleus. In nuclear physics, the convention is to designate a nucleus X as ${}^{A}_{Z}\mathrm{X}$, where A is its atomic mass number and Z its atomic number; for example, the ^{60}Co nucleus is identified as ${}^{60}_{27}\mathrm{Co}$ and the ^{226}Ra nucleus as ${}^{226}_{88}\mathrm{Ra}$. The atomic number Z is often omitted in references to an atom because the atom is already identified by its 1–3 letter symbol. In ion physics, the convention is to designate ions with + or – superscripts. For example, ${}^{4}_{2}\mathrm{He}^{+}$ stands for a singly ionized helium atom and ${}^{4}_{2}\mathrm{He}^{2+}$stands for a doubly ionized helium atom, also known as the α particle. With regard to relative values of atomic number Z and atomic mass number A of nuclei, the following conventions apply:

- An element may be composed of atoms that all have the same number of protons, i.e. have the same atomic number Z, but have a different number of neutrons (have different atomic mass numbers A). Such atoms of identical Z but differing A are called isotopes of a given element.
- The term ‘isotope’ is often misused to designate nuclear species. For example, ^{60}Co, ^{137}Cs and ^{226}Ra are not isotopes, since they do not belong to the same element. Rather than isotopes, they should be referred to as nuclides. On the other hand, it is correct to state that deuterium (with a nucleus called deuteron) and tritium (with a nucleus called triton) are heavy isotopes of hydrogen or that ^{59}Co and ^{60}Co are isotopes of cobalt. Thus, the term ‘radionuclide’ should be used to designate radioactive species; however, the term ‘radioisotope’ is often used for this purpose.
- A nuclide is an atomic species characterized by its nuclear composition (A, Z and the arrangement of nucleons within the nucleus). The term ‘nuclide’ refers to all atomic forms of all elements. The term ‘isotope’ is narrower and only refers to various atomic forms of a given chemical element.

In addition to being classified into isotopic groups (common atomic number Z), nuclides are also classified into groups with a common atomic mass number A (isobars) and a common number of neutrons (isotones). For example, ^{60}Co and ^{60}Ni are isobars with 60 nucleons each ($A = 60$), and ${}^{67}_{31}\mathrm{Ga}$, ${}^{67}_{32}\mathrm{Ge}$ and ${}^{67}_{33}\mathrm{As}$ are isobars with atomic mass number 67, while ^{3}H (tritium) and ^{4}He are isotones with two neutrons each ($A - Z = 2$), and ${}^{12}_{6}\mathrm{C}$, ${}^{13}_{7}\mathrm{N}$ and ${}^{14}_{8}\mathrm{O}$ are isotones with six neutrons each.

A tool for remembering these definitions is as follows: isoto**p**es have the same number of **p**rotons Z; isoto**n**es have the same number of **n**eutrons, $A - Z$; isob**a**rs have the same mass number $\boldsymbol{A}$.

If a nucleus exists in an excited state for some time, it is said to be in an isomeric (metastable) state. Isomers are, thus, nuclear species that have a common atomic number Z and a common atomic mass number A. For example, ^{99m}Tc is an isomeric state of ^{99}Tc and ^{60m}Co is an isomeric state of ^{60}Co.

1.3.1. Nuclear radius

The radius R of a nucleus with atomic mass number A is estimated from the following expression:

$$R = R_0\sqrt[3]{A} \tag{1.4}$$

where R_0 is the nuclear radius constant equal to 1.25 fm. Since the range of A in nature is from 1 to about 250, nuclear radius ranges from about 1 fm for a proton to about 8 fm for heavy nuclei.

1.3.2. Nuclear binding energy

The sum of the masses of the individual components of a nucleus that contains Z protons and $(A - Z)$ neutrons is larger than the actual mass of the nucleus. This difference in mass is called the mass defect (deficit) Δm and its energy equivalent Δmc^2 is called the total binding energy E_B of the nucleus. The total binding energy E_B of a nucleus can, thus, be defined as the energy liberated when Z protons and $(A - Z)$ neutrons are brought together to form the nucleus.

The binding energy per nucleon (E_B/A) in a nucleus (i.e. the total binding energy of a nucleus divided by the number of nucleons in the given nucleus) varies with the number of nucleons A and is of the order of ~8 MeV/nucleon.

A plot of the binding energy per nucleon E_B/A in megaelectronvolts per nucleon against the atomic mass number in the range from 1 to 250 is given in Fig. 1.1 and shows a rapid rise in E_B/A at small atomic mass numbers, a broad maximum of about 8.7 MeV/nucleon around $A \approx 60$ and a gradual decrease in E_B/A at large A. The larger the binding energy per nucleon (E_B/A) of an atom, the larger is the stability of the atom. Thus, the most stable nuclei in nature are the ones with $A \approx 60$ (iron, cobalt, nickel). Nuclei of light elements (small A) are generally less stable than nuclei with $A \approx 60$, and the heaviest nuclei (large A) are also less stable than nuclei with $A \approx 60$.

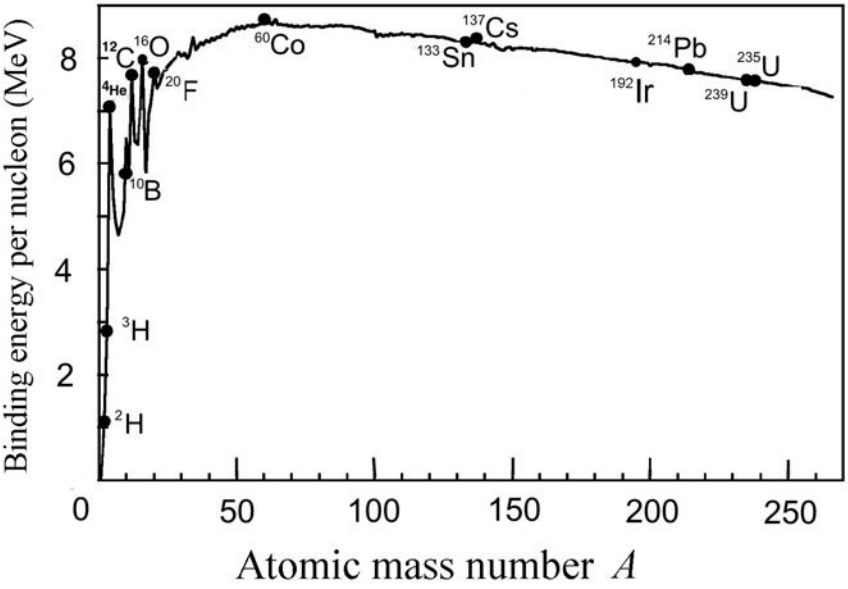

FIG. 1.1. Binding energy per nucleon in megaelectronvolts per nucleon against atomic mass number A. Data are from the National Institute of Science and Technology (NIST).

1.3.3. Nuclear fusion and fission

The peculiar shape of the E_B/A versus A curve (Fig. 1.1) suggests two methods for converting mass into energy: (i) fusion of nuclei at low A and (ii) fission of nuclei at large A:

- Fusion of two nuclei of very small mass, e.g. ${}^{2}_{1}\mathrm{H} + {}^{3}_{1}\mathrm{H} \rightarrow {}^{4}_{2}\mathrm{He} + \mathrm{n}$, will create a more massive nucleus and release a certain amount of energy. Experiments using controlled nuclear fusion for production of energy have so far not been successful in generating a net energy gain, i.e. the amount of energy consumed is still larger than the amount created. However, fusion remains an active field of research and it is reasonable to expect that in the future controlled fusion will play an important role in the production of electrical power.
- Fission attained by bombardment of certain elements of large mass (such as ^{235}U) by thermal neutrons in a nuclear reactor will create two lower mass and more stable nuclei, and transform some mass into kinetic energy of the two product nuclei. Hahn, Strassman, Meitner and Frisch described fission in 1939, and, in 1942, Fermi and colleagues at the University of Chicago carried out the first controlled chain reaction based on nuclear fission.

Since then, fission reactors have become an important means of production of electrical power.

1.3.4. Two-particle collisions and nuclear reactions

A common problem in nuclear physics and radiation dosimetry is the collision of two particles in which a projectile with mass m_1, velocity υ_1 and kinetic energy $(E_K)_1$ strikes a stationary target with mass m_2 and velocity $\upsilon_2 = 0$. The probability or cross-section for a particular collision as well as the collision outcome depends on the physical properties of the projectile (mass, charge, velocity, kinetic energy) and the stationary target (mass, charge).

As shown schematically in Fig. 1.2, the collision between the projectile and the target in the most general case results in an intermediate compound that subsequently decays into two reaction products: one of mass m_3 ejected with velocity υ_3 at an angle θ to the incident projectile direction, and the other of mass m_4 ejected with velocity υ_4 at an angle ϕ to the incident projectile direction.

Two-particle collisions are classified into three categories: (a) elastic scattering, (b) inelastic collisions and (c) nuclear reactions:

(a) Elastic scattering is a special case of a two-particle collision in which the products after the collision are identical to the products before collision, i.e. $m_3 = m_1$ and $m_4 = m_2$, and the total kinetic energy and momentum before the collision are equal to the total kinetic energy and momentum, respectively, after the collision.

(b) In inelastic scattering of a projectile m_1 on the target m_2, similarly to elastic scattering, the reaction products after collision are identical to the initial products, i.e. $m_3 = m_1$ and $m_4 = m_2$; however, the incident projectile transfers a portion of its kinetic energy to the target in the form of not only kinetic energy but also intrinsic excitation energy E^*.

(c) During a nuclear reaction, a collision between a projectile m_1 and a target m_2 takes place and will result in the formation of two reaction products m_3 and m_4, with the products having new atomic numbers. This process is shown schematically in Fig. 1.2. In any nuclear reaction, a number of physical quantities must be conserved, most notably charge, linear momentum and mass–energy. In addition, the sum of atomic numbers Z and the sum of atomic mass numbers A before and after the collision must also be conserved.

The Q value of a nuclear reaction is defined as the difference between the total rest energy before the reaction $(m_1c^2 + m_2c^2)$ and the total rest energy after the reaction $(m_3c^2 + m_4c^2)$ or:

$$Q = (m_1c^2 + m_2c^2) - (m_3c^2 + m_4c^2) \tag{1.5}$$

Each two-particle collision possesses a characteristic Q value that can be either positive, zero or negative. For $Q > 0$, the collision is termed 'exothermic' (also called exoergic) and results in a release of energy; for $Q = 0$, the collision is termed 'elastic' and for $Q < 0$, the collision is termed 'endothermic' (also called endoergic), and to take place, it requires an energy transfer from the projectile to the target. An exothermic reaction can occur spontaneously, while an endothermic reaction cannot take place unless the projectile has kinetic energy exceeding the threshold energy $(E_\mathrm{K})_\mathrm{thr}$ given as:

$$(E_\mathrm{K})_\mathrm{thr} = \frac{(m_3c^2 + m_4c^2)^2 - (m_1c^2 + m_2c^2)^2}{2m_2c^2} \approx -Q\left(1 + \frac{m_1}{m_2}\right) \tag{1.6}$$

where m_1c^2, m_2c^2, m_3c^2 and m_4c^2 are the rest energies of the projectile m_1, target m_2 and reaction products m_3 and m_4, respectively.

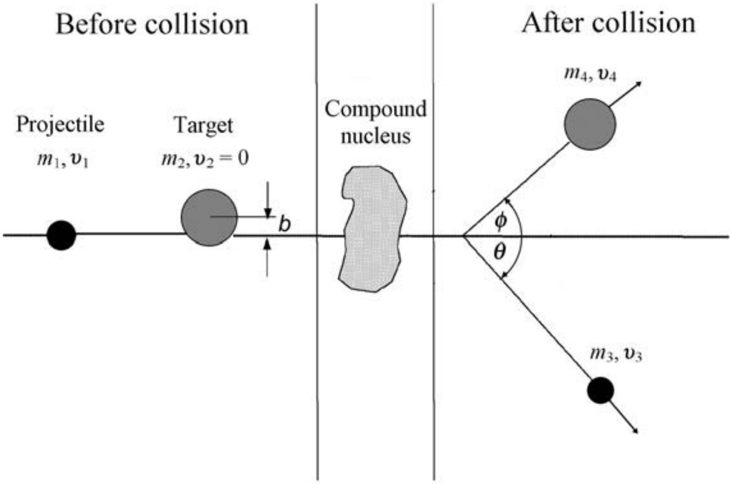

FIG. 1.2. Schematic representation of a two-particle collision of a projectile (incident particle) of mass m_1 and velocity υ_1 striking a stationary target with mass m_2 and velocity $\upsilon_2 = 0$. An intermediate compound entity is formed temporarily that subsequently decays into two reaction products of mass m_3 and m_4.

1.4. RADIOACTIVITY

Radioactivity, also known as radioactive decay, nuclear decay, nuclear disintegration and nuclear transformation, is a spontaneous process by which an unstable parent nucleus emits a particle or electromagnetic radiation and transforms into a more stable daughter nucleus that may or may not be stable. The unstable daughter nucleus will decay further in a decay series until a stable nuclear configuration is reached. Radioactive decay is usually accompanied by emission of energetic particles or γ ray photons or both.

All radioactive decay processes are governed by the same general formalism that is based on the definition of the activity $\mathcal{A}(t)$ and on a characteristic parameter for each radioactive decay process, the radioactive decay constant λ with dimensions of reciprocal time, usually in s^{-1}. The main characteristics of radioactive decay are as follows:

- The radioactive decay constant λ multiplied by a time interval that is much smaller than $1/\lambda$ represents the probability that any particular atom of a radioactive substance containing a large number $N(t)$ of identical radioactive atoms will decay (disintegrate) in that time interval. An assumption is made that λ is independent of the physical environment of a given atom.
- The activity $\mathcal{A}(t)$ of a radioactive substance containing a large number $N(t)$ of identical radioactive atoms represents the total number of decays (disintegrations) per unit time and is defined as a product between $N(t)$ and λ, i.e.:

$$\mathcal{A}(t) = \lambda N(t) \tag{1.7}$$

The SI unit of activity is the becquerel (Bq) given as 1 Bq = 1 s^{-1}. The becquerel and hertz both correspond to s^{-1}, but hertz refers to the frequency of periodic motion, while becquerel refers to activity.

The old unit of activity, the curie (Ci), was initially defined as the activity of 1 g of ^{226}Ra; 1 Ci $\cong 3.7 \times 10^{10}$ s^{-1}.

Subsequently, the activity of 1 g of ^{226}Ra was determined to be 3.665×10^{10} s^{-1}; however, the definition of the activity unit curie (Ci) was kept as 1 Ci = 3.7×10^{10} s^{-1}. Since the unit of activity the becquerel is 1 s^{-1}, the SI unit becquerel (Bq) and the old unit curie (Ci) are related as follows: 1 Ci = 3.7×10^{10} Bq and, consequently, 1 Bq = $(3.7 \times 10^{10})^{-1}$ Ci = 2.703×10^{-11} Ci.

Specific activity a is defined as activity $\mathcal{A}$ per unit mass m, i.e.:

$$a = \frac{\mathcal{A}}{m} = \frac{\lambda N}{m} = \frac{\lambda N_\mathrm{A}}{A} \tag{1.8}$$

where N_A is Avogadro's number.

Specific activity a of a radioactive atom depends on the decay constant λ and on the atomic mass number A of the radioactive atom. The units of specific activity are Bq/kg (SI unit) and Ci/g (old unit).

1.4.1. Decay of radioactive parent into a stable or unstable daughter

The simplest form of radioactive decay involves a radioactive parent nucleus P decaying with decay constant λ_P into a stable or unstable daughter nucleus D:

$$\mathrm{P} \xrightarrow{\lambda_\mathrm{P}} \mathrm{D} \tag{1.9}$$

The rate of depletion of the number of radioactive parent nuclei $N_\mathrm{P}(t)$ is equal to activity $\mathcal{A}_\mathrm{P}(t)$ at time t defined as the product $\lambda N(t)$ in Eq. (1.7). We, thus, have the following expression:

$$\frac{\mathrm{d}N_\mathrm{P}(t)}{\mathrm{d}t} = -\mathcal{A}_\mathrm{P}(t) = -\lambda_\mathrm{P} N_\mathrm{P}(t) \tag{1.10}$$

The fundamental differential equation in Eq. (1.10) for $N_\mathrm{P}(t)$ can be rewritten in general integral form:

$$\int\limits_{N_\mathrm{P}(0)}^{N_\mathrm{P}(t)} \frac{\mathrm{d}N_\mathrm{P}(t)}{N_\mathrm{P}} = -\int\limits_0^t \lambda_\mathrm{P}\, \mathrm{d}t \tag{1.11}$$

where $N_\mathrm{P}(0)$ is the initial condition represented by the number of radioactive nuclei at time $t = 0$.

Assuming that λ_P is constant, Eq. (1.11) can be solved to obtain:

$$\ln \frac{N_P(t)}{N_P(0)} = -\lambda_P t \tag{1.12}$$

or

$$N_P(t) = N_P(0) e^{-\lambda_P t} \tag{1.13}$$

Based on the definition of activity given in Eq. (1.7), the activity of parent nuclei P at time t can be expressed as follows:

$$\mathcal{A}_P(t) = \lambda_P N_P(t) = \lambda_P N_P(0) e^{-\lambda_P t} = \mathcal{A}_P(0) e^{-\lambda_P t} \tag{1.14}$$

where $\mathcal{A}_P(0) = \lambda_P N_P(0)$ is the initial activity of the radioactive substance.

The decay law of Eq. (1.14) applies to all radioactive nuclides irrespective of their mode of decay; however, the decay constant λ_P is different for each parent radioactive nuclide P and is the most important defining characteristic of a radioactive nuclide.

Two special time periods called half-life $(T_{1/2})_P$ and mean or average life τ_P are used to characterize a given radioactive parent substance P. The half-life $(T_{1/2})_P$ of a radioactive substance P is the time during which the number of radioactive nuclei of the substance decays to half of the initial value $N_P(0)$ present at time $t = 0$. It can also be stated that in the time of one half-life the activity $\mathcal{A}_P(t)$ of a radioactive substance decreases to one half of its initial value $\mathcal{A}_P(0) = \lambda_P N_P(0)$:

$$N_P[t = (T_{1/2})_P] = \frac{1}{2} N_P(0) = N_P(0) e^{-\lambda_P (T_{1/2})_P} \tag{1.15}$$

and

$$\mathcal{A}_P[t = (T_{1/2})_P] = \frac{1}{2} \mathcal{A}_P(0) = \mathcal{A}_P(0) e^{-\lambda_P (T_{1/2})_P} \tag{1.16}$$

From Eqs (1.15) and (1.16), it is noted that $e^{-\lambda_P (T_{1/2})_P}$ must equal 1/2, resulting in the following relationship between the decay constant λ_P and half-life $(T_{1/2})_P$:

$$\lambda_P = \frac{\ln 2}{(T_{1/2})_P} = \frac{0.693}{(T_{1/2})_P} \tag{1.17}$$

Mean (average) life τ_P of a radioactive parent P is defined as the time required for the number N_P of radioactive atoms or its activity $\mathcal{A}_P$ to fall to 1/e = 0.368 (or 36.8%) of the initial number of nuclei $N_P(0)$ or of the initial activity $\mathcal{A}_P(0)$, respectively. Thus, the following expressions describe the mean half-life:

$$N_P(t=\tau_P)=\frac{1}{e}N_P(0)=0.368N_P(0)=N_P(0)e^{-\lambda_P\tau_P} \tag{1.18}$$

and

$$\mathcal{A}_P(t=\tau_P)=\frac{1}{e}\mathcal{A}_P(0)=0.368\mathcal{A}_P(0)=\mathcal{A}_P(0)e^{-\lambda_P\tau_P} \tag{1.19}$$

From Eqs (1.18) and (1.19), it is noted that $e^{-\lambda_P\tau_P}$ must be equal to $1/e = e^{-1} = 0.368$, resulting in $\lambda_P\tau_P = 1$ and $\tau_P = 1/\lambda_P$. We now get the following relationship between mean life τ_P and half-life $(T_{1/2})_P$ using Eq. (1.17) and $\tau_P = 1/\lambda_P$:

$$\lambda_P=\frac{\ln\ 2}{(T_{1/2})_P}=\frac{1}{\tau_P} \tag{1.20}$$

and

$$\tau_P=\frac{(T_{1/2})_P}{\ln\ 2}=1.44(T_{1/2})_P \tag{1.21}$$

A typical example of a radioactive decay for initial condition $\mathcal{A}_P(t = 0) = \mathcal{A}_P(0)$ is shown in Fig. 1.3 with a plot of parent activity $\mathcal{A}_P(t)$ against time t given in Eq. (1.14).

1.4.2. Radioactive series decay

The radioactive decay of parent P into stable daughter D, discussed in Section 1.4.1, is the simplest known radioactive decay process; however, the decay of a radioactive parent P with decay constant λ_P into a radioactive (unstable) daughter D which in turn decays with decay constant λ_D into a stable or unstable grand-daughter G, i.e. $(P \xrightarrow{\lambda_P} D \xrightarrow{\lambda_D} G)$, is much more common and results in a radioactive decay series for which the last decay product is stable.

The parent P in the decay series follows a straightforward radioactive decay described by Eq. (1.16) for the rate of change of the number of parent

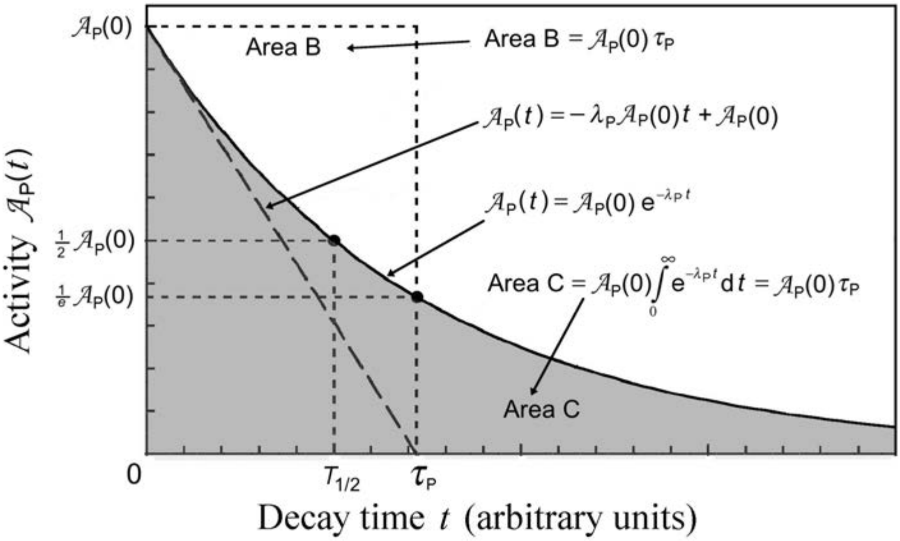

FIG. 1.3. Activity $\mathcal{A}_P(t)$ plotted against time t for a simple decay of a radioactive parent P into a stable or unstable daughter D. The concepts of half-life $(T_{1/2})_P$ and mean life τ_P are also illustrated. The area under the exponential decay curve from t = 0 to t = ∞ is equal to the product $\mathcal{A}_P(0)\tau_P$ where $\mathcal{A}_P(0)$ is the initial activity of the parent P. The slope of the tangent to the decay curve at t = 0 is equal to $\lambda_P\mathcal{A}_P(0)$ and this tangent crosses the abscissa axis at $t = \tau_P$.

nuclei $\mathrm{d}N_\mathrm{P}(t)/\mathrm{d}t$. The rate of change of the number of daughter nuclei $\mathrm{d}N_\mathrm{D}(t)/\mathrm{d}t$, however, is more complicated and consists of two components, one being the supply of new daughter nuclei D through the decay of P given as $\lambda_\mathrm{P}N_\mathrm{P}(t)$ and the other being the loss of daughter nuclei D from the decay of D to G given as $-\lambda_\mathrm{D}N_\mathrm{D}(t)$, resulting in the following expression for $\mathrm{d}N_\mathrm{D}(t)/\mathrm{d}t$:

$$\frac{\mathrm{d}N_\mathrm{D}(t)}{\mathrm{d}t} = \lambda_\mathrm{P}N_\mathrm{P}(t) - \lambda_\mathrm{D}N_\mathrm{D}(t) = \lambda_\mathrm{P}N_\mathrm{P}(0)\mathrm{e}^{-\lambda_\mathrm{P}t} - \lambda_\mathrm{D}N_\mathrm{D}(t) \qquad (1.22)$$

With the initial conditions for time $t = 0$ assuming that (i) the initial number of parent nuclei P is $N_\mathrm{P}(t = 0) = N_\mathrm{P}(0)$, and (ii) there are no daughter D nuclei present, i.e. $N_\mathrm{D}(t = 0) = 0$, the solution of the differential equation in Eq. (1.22) reads as follows:

$$N_\mathrm{D}(t) = N_\mathrm{P}(0)\frac{\lambda_\mathrm{P}}{\lambda_\mathrm{D} - \lambda_\mathrm{P}}\left[\mathrm{e}^{-\lambda_\mathrm{P}t} - \mathrm{e}^{-\lambda_\mathrm{D}t}\right] \qquad (1.23)$$

Recognizing that the activity of the daughter $\mathcal{A}_D(t)$ is $\lambda_D N_D(t)$, the daughter activity $\mathcal{A}_D(t)$ is written as:

$$\begin{aligned}\mathcal{A}_D(t) &= N_P(0)\frac{\lambda_D\lambda_P}{\lambda_D-\lambda_P}\left[e^{-\lambda_P t}-e^{-\lambda_D t}\right] = \mathcal{A}_P(0)\frac{\lambda_D}{\lambda_D-\lambda_P}\left[e^{-\lambda_P t}-e^{-\lambda_D t}\right]\\ &= \mathcal{A}_P(0)\frac{1}{1-\frac{\lambda_P}{\lambda_D}}\left[e^{-\lambda_P t}-e^{-\lambda_D t}\right] = \mathcal{A}_P(t)\frac{\lambda_D}{\lambda_D-\lambda_P}\left[1-e^{-(\lambda_D-\lambda_P)t}\right]\end{aligned} \tag{1.24}$$

where

$\mathcal{A}_D(t)$ is the activity at time t of the daughter nuclei equal to $\lambda_D N_D(t)$;
$\mathcal{A}_P(0)$ is the initial activity of the parent nuclei present at time $t = 0$;

and $\mathcal{A}_P(t)$ is the activity at time t of the parent nuclei equal to $\lambda_P N_P(t)$.

While for initial conditions $\mathcal{A}_P(t = 0) = \mathcal{A}_P(0)$ and $\mathcal{A}_D(t = 0) = 0$, the parent P activity $\mathcal{A}_P(t)$ follows the exponential decay law of Eq. (1.14) shown in Fig. 1.3, the daughter D activity $\mathcal{A}_D(t)$ starts at 0, then initially rises with time t, reaches a maximum at a characteristic time $t = (t_{\max})_D$, and then diminishes to reach 0 at $t = \infty$. The characteristic time $(t_{\max})_D$ is given as follows:

$$(t_{\max})_D = \frac{\ln\frac{\lambda_P}{\lambda_D}}{\lambda_P-\lambda_D} \tag{1.25}$$

1.4.3. Equilibrium in parent–daughter activities

In many parent P → daughter D → grand-daughter G relationships, after a certain time t the parent and daughter activities reach a constant ratio independent of a further increase in time t. This condition is referred to as radioactive equilibrium and can be analysed by examining the behaviour of the activity ratio $\mathcal{A}_D(t)/\mathcal{A}_P(t)$ obtained from Eq. (1.24) as:

$$\frac{\mathcal{A}_D(t)}{\mathcal{A}_P(t)} = \frac{\lambda_D}{\lambda_D-\lambda_P}\left[1-e^{-(\lambda_D-\lambda_P)t}\right] = \frac{1}{1-\frac{\lambda_P}{\lambda_D}}\left[1-e^{-(\lambda_D-\lambda_P)t}\right] \tag{1.26}$$

Three possibilities merit special consideration:

(a) The half-life of the daughter exceeds that of the parent: $(T_{1/2})_D > (T_{1/2})_P$ resulting in $\lambda_D < \lambda_P$. The activity ratio $\mathcal{A}_D(t)/\mathcal{A}_P(t)$ of Eq. (1.26) is written as:

$$\frac{\mathcal{A}_D(t)}{\mathcal{A}_P(t)} = \frac{\lambda_D}{\lambda_P - \lambda_D}\left[e^{(\lambda_P - \lambda_D)t} - 1 \right] \tag{1.27}$$

The ratio $\mathcal{A}_D(t)/\mathcal{A}_P(t)$ increases exponentially with time t, indicating that no equilibrium between the parent activity $\mathcal{A}_P(t)$ and daughter activity $\mathcal{A}_D(t)$ will be reached.

(b) The half-life of the daughter is shorter than that of the parent: $(T_{1/2})_D < (T_{1/2})_P$ or $\lambda_D > \lambda_P$.
The activity ratio $\mathcal{A}_D(t)/\mathcal{A}_P(t)$ at large t becomes a constant equal to $\lambda_D/(\lambda_D - \lambda_P)$ and is then independent of t and larger than unity, implying transient equilibrium, i.e.:

$$\frac{\mathcal{A}_D(t)}{\mathcal{A}_P(t)} = \frac{\lambda_D}{\lambda_D - \lambda_P} = \text{const} > 1 \tag{1.28}$$

(c) The half-life of the daughter is much shorter than that of the parent: $(T_{1/2})_D \ll (T_{1/2})_P$ or $\lambda_D \gg \lambda_P$.
For relatively large time $t \gg t_{max}$, the activity ratio $\mathcal{A}_D(t)/\mathcal{A}_P(t)$ of Eq. (1.28) simplifies to:

$$\frac{\mathcal{A}_D(t)}{\mathcal{A}_P(t)} \approx 1 \tag{1.29}$$

The activity of the daughter $\mathcal{A}_D(t)$ very closely approximates that of its parent $\mathcal{A}_P(t)$, i.e. $\mathcal{A}_D(t) \approx \mathcal{A}_P(t)$, and they decay together at the rate of the parent. This special case of transient equilibrium in which the daughter and parent activities are essentially identical is called secular equilibrium.

1.4.4. Production of radionuclides (nuclear activation)

In 1896, Henri Becquerel discovered natural radioactivity, and in 1934 Frédèric Joliot and Irène Curie-Joliot discovered artificial radioactivity. Most natural radionuclides are produced through one of four radioactive decay chains, each chain fed by a long lived and heavy parent radionuclide. The vast majority of currently known radionuclides, however, are human-made and artificially produced through a process of nuclear activation which uses bombardment of a stable nuclide with a suitable energetic particle or high energy photons to induce a nuclear transformation. Various particles or electromagnetic radiation generated by a variety of machines are used for this purpose, most notably neutrons from nuclear reactors for neutron activation, protons from cyclotrons or synchrotrons for proton activation, and X rays from high energy linear accelerators for nuclear photoactivation.

Neutron activation is important in production of radionuclides used for external beam radiotherapy, brachytherapy, therapeutic nuclear medicine and nuclear medicine imaging also referred to as molecular imaging; proton activation is important in production of positron emitters used in positron emission tomography (PET) imaging; and nuclear photoactivation is important from a radiation protection point of view when components of high energy radiotherapy machines become activated during patient treatment and pose a potential radiation risk to staff using the equipment.

A more in depth discussion of radionuclide production can be found in Chapter 4.

1.4.5. Modes of radioactive decay

Nucleons are bound together to form the nucleus by the strong nuclear force that, in comparison to the proton–proton Coulomb repulsive force, is at least two orders of magnitude larger but of extremely short range (only a few femtometres). To bind the nucleons into a stable nucleus, a delicate equilibrium between the number of protons and the number of neutrons must exist. For light (low A) nuclear species, a stable nucleus is formed by an equal number of protons and neutrons ($Z = N$). Above the nucleon number $A \approx 40$, more neutrons than protons must constitute the nucleus to form a stable configuration in order to overcome the Coulomb repulsion among the charged protons.

If the optimal equilibrium between protons and neutrons does not exist, the nucleus is unstable (radioactive) and decays with a specific decay constant λ into a more stable configuration that may also be unstable and decay further, forming a decay chain that eventually ends with a stable nuclide.

Radioactive nuclides, either naturally occurring or artificially produced by nuclear activation or nuclear reactions, are unstable and strive to reach more stable nuclear configurations through various processes of spontaneous radioactive decay that involve transformation to a more stable nuclide and emission of energetic particles. General aspects of spontaneous radioactive decay may be discussed using the formalism based on the definitions of activity $\mathcal{A}$ and decay constant λ without regard for the actual microscopic processes that underlie the radioactive disintegrations.

A closer look at radioactive decay processes shows that they are divided into six categories, consisting of three main categories of importance to medical use of radionuclides and three categories of less importance. The main categories are: (i) alpha (α) decay, (ii) beta (β) decay encompassing three related decay processes (beta minus, beta plus and electron capture) and (iii) gamma (γ) decay encompassing two competing decay processes (pure γ decay and internal conversion). The three less important radioactive decay categories are: (i) spontaneous fission, (ii) proton emission decay and (iii) neutron emission decay.

Nuclides with an excess number of neutrons are referred to as neutron-rich; nuclides with an excess number of protons are referred to as proton-rich. The following features are notable:

- For a slight proton–neutron imbalance in the nucleus, radionuclides decay by β decay characterized by transformation of a proton into a neutron in β^+ decay, and transformation of a neutron into a proton in β^- decay.
- For a large proton–neutron imbalance in the nucleus, the radionuclides decay by emission of nucleons: α particles in α decay, protons in proton emission decay and neutrons in neutron emission decay.
- For very large atomic mass number nuclides ($A > 230$), spontaneous fission, which competes with α decay, is also possible.

Excited nuclei decay to their ground state through γ decay. Most of these transformations occur immediately upon production of the excited state by either α or β decay; however, a few exhibit delayed decays that are governed by their own decay constants and are referred to as metastable states (e.g. ^{99m}Tc).

Nuclear transformations are usually accompanied by emission of energetic particles (charged particles, neutral particles, photons, etc.). The particles released in the various decay modes are as follows:

- Alpha particles in α decay;
- Electrons in β^- decay;
- Positrons in β^+ decay;
- Neutrinos in β^+ decay;
- Antineutrinos in β^- decay;
- Gamma rays in γ decay;
- Atomic orbital electrons in internal conversion;
- Neutrons in spontaneous fission and in neutron emission decay;
- Heavier nuclei in spontaneous fission;
- Protons in proton emission decay.

In each nuclear transformation, a number of physical quantities must be conserved. The most important of these quantities are: (i) total energy, (ii) momentum, (iii) charge, (iv) atomic number and (v) atomic mass number (number of nucleons).

The total energy of particles released by the transformation process is equal to the net decrease in the rest energy of the neutral atom, from parent P to daughter D. The disintegration (decay) energy, often referred to as the Q value for the radioactive decay, is defined as follows:

$$Q = \{M(\mathrm{P}) - [M(\mathrm{D}) + m]\}c^2 \tag{1.30}$$

where M(P), M(D) and m are the nuclear rest masses (in unified atomic mass units u) of the parent, daughter and emitted particles, respectively.

For radioactive decay to be energetically possible, the Q value must be greater than zero. This means that spontaneous radioactive decay processes release energy and are called exoergic or exothermic. For $Q > 0$, the energy equivalent of the Q value is shared as kinetic energy between the particles emitted in the decay process and the daughter product. Since the daughter generally has a much larger mass than the other emitted particles, the kinetic energy acquired by the daughter is usually negligibly small.

1.4.6. Alpha decay

In α decay, a radioactive parent nucleus P decays into a more stable daughter nucleus D by ejecting an energetic α particle. Since the α particle is a ^{4}He nucleus ($^4_2\mathrm{He}^{2+}$), in α decay the parent's atomic number Z decreases by two and its atomic mass number A decreases by four:

$$^A_Z\mathrm{P} \rightarrow {}^{A-4}_{Z-2}\mathrm{D} + {}^4_2\mathrm{He}^{2+} = {}^{A-4}_{Z-2}\mathrm{D} + \alpha \tag{1.31}$$

Naturally occurring α particles have kinetic energies between 4 and 9 MeV; their range in air is between 1 and 10 cm, and their range in tissue is between 10 and 100 µm.

Typical examples of α decay are the decay of ^{226}Ra with a half-life of 1602 a into ^{222}Rn which is also radioactive and decays with a half-life of 3.82 d into ^{218}Po:

$$^{226}_{88}\mathrm{Ra} \xrightarrow[T_{1/2}=1602\ \mathrm{a}]{} {}^{222}_{86}\mathrm{Rn} + \alpha$$

and

$$^{222}_{86}\mathrm{Rn} \xrightarrow[T_{1/2}=3.82\ \mathrm{d}]{} {}^{218}_{84}\mathrm{Po} + \alpha \tag{1.32}$$

1.4.7. Beta minus decay

In beta minus (β^-) decay, a neutron-rich parent nucleus P transforms a neutron into a proton and ejects an electron e^- and an electronic antineutrino $\bar{\nu}_e$. Thus, in β^- decay, the atomic number of the daughter increases by one, i.e. $Z_D = Z_P + 1$, the atomic mass number remains constant, i.e. $A_D = A_P$, and the general relationship for β^- decay is given as:

$$^{A}_{Z}\mathrm{P} \rightarrow {}_{Z+1}^{A}\mathrm{D} + \mathrm{e}^- + \bar{\nu}_\mathrm{e} \tag{1.33}$$

A typical example of β^- decay is the decay of ^{60}Co with a half-life of 5.26 a into an excited state of ^{60}Ni ($^{60}_{28}\mathrm{Ni}^*$). Excited states of ^{60}Ni progress to the ground state of ^{60}Ni instantaneously (within 10^{-12} s) through emission of γ rays in γ decay:

$$^{60}_{27}\mathrm{Co} \xrightarrow[T_{1/2}=5.26\ \mathrm{a}]{} {}^{60}_{28}\mathrm{Ni}^* + \mathrm{e}^- + \bar{\nu}_\mathrm{e} \tag{1.34}$$

1.4.8. Beta plus decay

In beta plus (β^+) decay, a proton-rich parent nucleus P transforms a proton into a neutron and ejects a positron e^+ and an electronic neutrino ν_e. Thus, in β^+ decay, the atomic number of the daughter decreases by one, i.e. $Z_D = Z_P - 1$, the atomic mass number, just as in β^- decay, remains constant, i.e. $A_D = A_P$, and the general relationship for β^+ decay is written as:

$$^{A}_{Z}\mathrm{P} \rightarrow {}_{Z-1}^{A}\mathrm{D} + \mathrm{e}^+ + \nu_\mathrm{e} \tag{1.35}$$

Radionuclides undergoing β^+ decay are often called positron emitters and are used in medicine for functional imaging with the special imaging technique

PET. The most common tracer for PET studies is fluorodeoxyglucose (FDG) labelled with ^{18}F which serves as a good example of β^+ decay:

$$^{18}_{9}\mathrm{F} \xrightarrow[T_{1/2}=110\ \mathrm{min}]{} {}^{18}_{8}\mathrm{O} + \mathrm{e}^+ + \nu_\mathrm{e} \tag{1.36}$$

1.4.9. Electron capture

Electron capture radioactive decay may occur when an atomic electron ventures inside the nuclear volume, is captured by a proton, triggers a proton to neutron transformation, and an ejection of an electronic neutrino ν_e. In electron capture, as in β^+ decay, the atomic number of the daughter decreases by one, i.e. $Z_\mathrm{D} = Z_\mathrm{P} - 1$, and its atomic mass number, just as in β^- and β^+ decay, remains constant, i.e. $A_\mathrm{D} = A_\mathrm{P}$. The general relationship for electron capture decay is written as:

$$^{A}_{Z}\mathrm{P} + \mathrm{e}^- \xrightarrow[T_{1/2}=60\ \mathrm{d}]{} {}^{\ \ A}_{Z-1}\mathrm{D} + \nu_\mathrm{e} \tag{1.37}$$

A simple example of electron capture decay is the decay of ^{125}I with a half-life of 60 d into an excited state of ^{125}Te which then decays to the ground state of ^{125}Te through γ decay and internal conversion:

$$^{125}_{53}\mathrm{I} + \mathrm{e}^- \xrightarrow[T_{1/2}=60\ \mathrm{d}]{} {}^{125}_{52}\mathrm{Te}^* + \nu_\mathrm{e} \tag{1.38}$$

1.4.10. Gamma decay and internal conversion

Alpha decay as well as the three β decay modes (β^-, β^+ and electron capture) may produce a daughter nucleus in an excited state without expending the full amount of the decay energy available. The daughter nucleus will then reach its ground state, either instantaneously or with some time delay (isomeric metastable state), through one of the following two processes:

(a) By emitting the excitation energy in the form of one or more γ photons in a decay process referred to as γ decay.

(b) By transferring the excitation energy to one of its associated atomic orbital electrons (usually a K shell electron) in a process called internal conversion. The vacancy left behind by the ejected orbital electron is filled by a transition from a higher atomic shell, resulting in characteristic X rays and/or Auger electrons.

In most radioactive α or β decays, the daughter nucleus de-excitation occurs instantaneously (i.e. within 10^{-12} s), so that the emitted γ rays are referred to as if they were produced by the parent nucleus (e.g. ^{60}Co γ rays). The γ rays produced from isomeric transitions are attributed to the isomeric daughter product (e.g. $^{99\text{m}}$Tc γ rays).

The γ decay process and the internal conversion process may be represented, respectively, as follows:

$$^{A}_{Z}\text{X}^{*} \rightarrow {}^{A}_{Z}\text{X} + \gamma \tag{1.39}$$

and

$$^{A}_{Z}\text{X}^{*} \rightarrow {}^{A}_{Z}\text{X}^{+} + \text{e}^{-} \rightarrow {}^{A}_{Z}\text{X} \tag{1.40}$$

where

$^{A}_{Z}\text{X}^{*}$ stands for an excited state of the nucleus $^{A}_{Z}\text{X}$;

and $^{A}_{Z}\text{X}^{+}$ is the singly ionized state of atom $^{A}_{Z}\text{X}$ following internal conversion decay.

An example of γ decay is the transition of an excited $^{60}_{28}\text{Ni}^{*}$ nucleus, resulting from the β^{-} decay of $^{60}_{27}\text{Co}$, into stable $^{60}_{28}\text{Ni}$ through an emission of two γ rays with energies of 1.17 and 1.33 MeV. An example of internal conversion decay is the decay of excited $^{125}_{52}\text{Te}^{*}$ which results from an electron capture decay of ^{125}I into stable $^{125}_{52}\text{Te}$ through emission of 35 keV γ rays (7 %) and internal conversion electrons (93%).

1.4.11. Characteristic (fluorescence) X rays and Auger electrons

A large number of radionuclides used in nuclear medicine (e.g. $^{99\text{m}}$Tc, ^{123}I, ^{201}Tl, ^{64}Cu) decay by electron capture and/or internal conversion. Both processes leave the atom with a vacancy in an inner atomic shell, most commonly the K shell. The vacancy in the inner shell is filled by an electron from a higher level atomic shell and the binding energy difference between the two shells is either emitted as a characteristic X ray (fluorescence photon) or transferred to a higher shell orbital electron which is then emitted from the atom as an Auger electron with a kinetic energy equal to the transferred energy minus the binding energy of the emitted Auger electron. Emission of characteristic photons and Auger electrons is discussed further in Section 1.6.4.

1.5. ELECTRON INTERACTIONS WITH MATTER

As an energetic charged particle, such as an electron or positron, traverses an absorbing medium, it experiences a large number of Coulomb interactions with the nuclei and orbital electrons of absorber atoms before its kinetic energy is expended. In each interaction, the charged particle's path may be altered (elastic or inelastic scattering) and it may lose some of its kinetic energy that will be transferred to the medium or to photons. Charged particle interactions with orbital electrons of the absorber result in collision (ionization loss); interactions with nuclei of the absorber result in radiation loss. Each of these possible interactions between the charged particle and absorber atom is characterized by a specific cross-section (probability) σ for the particular interaction. The energy loss of the charged particle propagating through an absorber depends on the properties of the particle, such as its mass, charge, velocity and energy, as well as on the properties of the absorber, such as its density and atomic number.

Stopping power is a parameter used to describe the gradual loss of energy of the charged particle as it penetrates into an absorbing medium. Two classes of stopping power are known: collision stopping power s_{col} results from charged particle interaction with orbital electrons of the absorber and radiation stopping power s_{rad} results from charged particle interaction with nuclei of the absorber. The total stopping power s_{tot} is the sum of the collision stopping power and the radiation stopping power.

1.5.1. Electron–orbital interactions

Coulomb interactions between the incident electron or positron and orbital electrons of an absorber result in ionizations and excitations of absorber atoms. Ionization is described as ejection of an orbital electron from the absorber atom, thereby producing an ion. Excitation, on the other hand, is defined as the transfer of an orbital electron of the absorber atom from an allowed orbit to a higher allowed orbit (shell), thereby producing an excited atom. Atomic excitations and ionizations result in collision energy loss and are characterized by collision (also known as ionization) stopping powers.

1.5.2. Electron–nucleus interactions

Coulomb interactions between the incident electron or positron and nuclei of the absorber atom result in particle scattering. The majority of these scattering events are elastic and result in no energy loss. However, when the scattering is inelastic, the incident charged particle loses part of its kinetic energy through production of X ray photons referred to as bremsstrahlung radiation. This energy loss is characterized by radiation stopping powers and is governed by the Larmor relationship which states that the rate of energy loss is proportional to the square of the particle's acceleration and the square of the particle's charge.

1.6. PHOTON INTERACTIONS WITH MATTER

1.6.1. Exponential absorption of photon beam in absorber

The most important parameter used in characterization of X ray or γ ray penetration into absorbing media is the linear attenuation coefficient μ. This coefficient depends on the energy $h\nu$ of the photon and the atomic number Z of the absorber, and may be described as the probability per unit path length that a photon will have an interaction with the absorber. The attenuation coefficient μ is determined experimentally by aiming a narrowly collimated monoenergetic photon beam $h\nu$ onto a suitable radiation detector and placing an absorber material of varying thickness x between the photon source and the detector. The absorber decreases the detector signal intensity from $I(x)$ which is measured with no absorber in the beam ($x = 0$) to $I(x)$ measured with an absorber of thickness $x > 0$ in the beam.

$\mathrm{d}I(x)/\mathrm{d}x$, the rate of change in beam intensity $I(x)$ transmitted through an absorber of thickness x, is equal to the product of the attenuation coefficient μ and the beam intensity $I(x)$ at thickness x (see Eq. (1.41)). Alternatively, it can be said that an absorber of thickness $\mathrm{d}x$ reduces the beam intensity by $\mathrm{d}I$ and the fractional reduction in intensity $-\mathrm{d}I/I$ is equal to the product of the attenuation coefficient μ and the absorber layer thickness $\mathrm{d}x$ (see Eq. (1.42)). The following expressions are obtained, respectively:

$$\frac{\mathrm{d}I(x)}{\mathrm{d}x} = -\mu I(x) \tag{1.41}$$

and

$$-\frac{\mathrm{d}I}{I} = \mu \,\mathrm{d}x \tag{1.42}$$

where the negative sign is used to indicate a decrease in signal $I(x)$ with an increase in absorber thickness x.

It should be noted that Eqs (1.41) and (1.42) can be considered identical.

The form of Eq. (1.41) is identical to the form of Eq. (1.10) that deals with simple radioactive decay; however, it must be noted that in radioactive decay the product $\lambda N(t)$ is defined as activity $\mathcal{A}(t)$, while in photon beam attenuation the product $\mu I(x)$ does not have a special name and symbol.

Integration of Eq. (1.42) over absorber thickness x from 0 to x and over intensity $I(x)$ from the initial intensity $I(0)$ (no absorber) to intensity $I(x)$ at absorber thickness x, gives:

$$\int_{I(0)}^{I(x)} \frac{\mathrm{d}I}{I} = -\int_{0}^{x} \mu \, \mathrm{d}x \tag{1.43}$$

resulting in:

$$I(x) = I(0)\mathrm{e}^{-\mu x} \tag{1.44}$$

where it is assumed that in a homogeneous absorber the attenuation coefficient μ is uniform and independent of absorber thickness x.

1.6.2. Characteristic absorber thicknesses

Equation (1.44) represents the standard expression for the exponential attenuation of a monoenergetic narrow photon beam. A typical exponential plot of intensity $I(x)$ against absorber thickness x of Eq. (1.44) is shown in Fig. 1.4 for a monoenergetic and narrow photon beam. The figure also defines three special absorber thicknesses used for characterization of photon beams: half-value layer (HVL), mean free path (MFP) and tenth-value layer (TVL):

— HVL (or $x_{1/2}$) is defined as the thickness of a homogeneous absorber that attenuates the narrow beam intensity $I(0)$ to one half (50%) of the original intensity, i.e. $I(x_{1/2}) = 0.5I(0)$. The relationship between the HVL $x_{1/2}$ and the attenuation coefficient μ is determined from the basic definition of the HVL as follows:

$$I(x_{1/2}) = 0.5I(0) = I(0)\mathrm{e}^{-\mu x_{1/2}} \tag{1.45}$$

resulting in:

$$\frac{1}{2} = \mathrm{e}^{-\mu x_{1/2}} \qquad \text{or} \qquad \mu x_{1/2} = \ln 2 = 0.693$$

and

$$\text{HVL} = x_{1/2} = \frac{\ln 2}{\mu} \tag{1.46}$$

- MFP (or $\bar{x}$) or relaxation length is the thickness of a homogeneous absorber that attenuates the beam intensity $I(0)$ to 1/e = 0.368 (36.8%) of its original intensity, i.e. $I(\bar{x}) = 0.368I(0)$. The photon MFP is the average distance a photon of energy $h\nu$ travels through a given absorber before undergoing an interaction. The relationship between the MFP $\bar{x}$ and the attenuation coefficient μ is determined from the basic definition of the MFP as follows:

$$I(\bar{x}) = \frac{1}{\text{e}} I(0) = 0.368 I(0) = I(0)\text{e}^{-\mu\bar{x}} \tag{1.47}$$

resulting in:

$$\frac{1}{\text{e}} = \text{e}^{-\mu\bar{x}} \qquad \text{or} \qquad \mu\bar{x} = 1$$

and

$$\text{MFP} = \bar{x} = \frac{1}{\mu} \tag{1.48}$$

- TVL (or $x_{1/10}$) is the thickness of a homogeneous absorber that attenuates the beam intensity $I(0)$ to one tenth (10%) of its original intensity, i.e. $I(x_{1/10}) = 0.1I(0)$. The relationship between the TVL $x_{1/10}$ and the attenuation coefficient μ is determined from the basic definition of the TVL as follows:

$$I(x_{1/10}) = 0.1 I(0) = I(0)\text{e}^{-\mu x_{1/10}} \tag{1.49}$$

resulting in:

$$\frac{1}{10} = \text{e}^{-\mu x_{1/10}} \qquad \text{or} \qquad \mu x_{1/10} = \ln 10 = 2.303$$

and

$$\mathrm{TVL} = x_{1/10} = \frac{\ln 10}{\mu} \tag{1.50}$$

From Eqs (1.46), (1.48) and (1.50), the linear attenuation coefficient μ may be expressed in terms of $x_{1/2}$, $\bar{x}$ and $x_{1/10}$, respectively, as follows:

$$\mu = \frac{\ln 2}{x_{1/2}} = \frac{1}{\bar{x}} = \frac{\ln 10}{x_{1/10}} \tag{1.51}$$

resulting in the following relationships among the characteristic thicknesses:

$$x_{1/2} = (\ln 2)\bar{x} = \frac{\ln 2}{\ln 10} x_{1/10} = 0.301 x_{1/10} \tag{1.52}$$

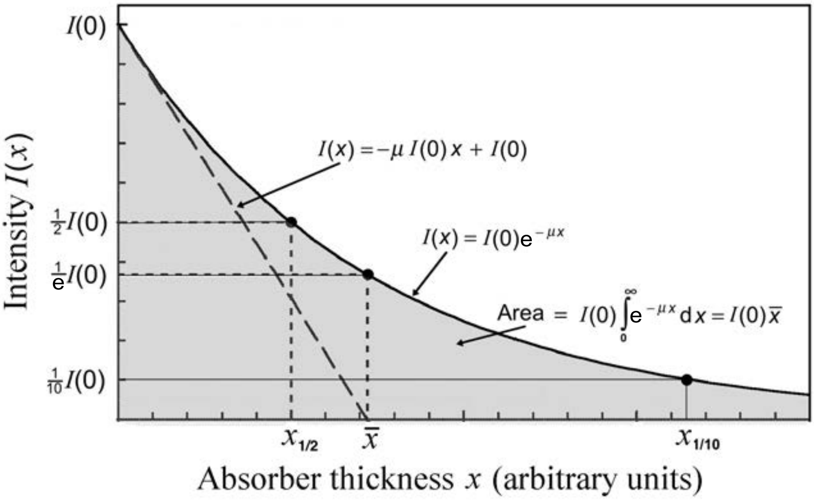

FIG. 1.4. Intensity I(x) against absorber thickness x for a monoenergetic photon beam. Half-value layer $x_{1/2}$, mean free path $\bar{x}$ and tenth-value layer $x_{1/10}$ are also illustrated. The area under the exponential attenuation curve from x = 0 to x = ∞ is equal to the product $I(0)\bar{x}$ where I(0) is the initial intensity of the monoenergetic photon beam. The slope of the tangent to the attenuation curve at x = 0 is equal to $\mu I(0)$ and this tangent crosses the abscissa (x) axis at $x = \bar{x}$.

1.6.3. Attenuation coefficients

In addition to the linear attenuation coefficient μ, three other related attenuation coefficients are in use for describing photon beam attenuation characteristics in absorbers: mass attenuation coefficient μ_{m}, atomic attenuation coefficient ${}_{\mathrm{a}}\mu$ and electronic attenuation coefficient ${}_{\mathrm{e}}\mu$. The attenuation coefficients are related as follows:

$$\mu = \rho\,\mu_{\mathrm{m}} = n^{\square}\,{}_{\mathrm{a}}\mu = Z\,n^{\square}\,{}_{\mathrm{e}}\mu \tag{1.53}$$

where

ρ is the mass density of the absorber;
$n^{\square}$ is the number of atoms N_{a} per volume V of the absorber, i.e. $n^{\square} = N_{\mathrm{a}}/V$, and $N_{\mathrm{a}}/V = \rho N_{\mathrm{a}}/m = \rho N_{\mathrm{A}}/A$ with m the mass of the absorber, N_{A} Avogadro’s number of atoms per mole and A the atomic mass of the absorber in grams per mole;
Z is the atomic number of the absorber;

and $Z\,n^{\square}$ is the number of electrons per unit volume of absorber, i.e. $Z\,n^{\square} = \rho Z N_{\mathrm{A}}/A$.

In radiation dosimetry, two energy-related coefficients are in use: (i) the energy transfer coefficient μ_{tr} that accounts for the mean energy transferred $\bar{E}_{\mathrm{tr}}$ from photons to charged particles (electrons and positrons) in a photon–atom interaction; and (ii) the energy absorption coefficient μ_{ab} that accounts for the mean energy absorbed $\bar{E}_{\mathrm{ab}}$ in the medium. The two coefficients are given as follows:

$$\mu_{\mathrm{tr}} = \mu\frac{\bar{E}_{\mathrm{tr}}}{h\nu} \tag{1.54}$$

and

$$\mu_{\mathrm{ab}} = \mu\frac{\bar{E}_{\mathrm{ab}}}{h\nu} \tag{1.55}$$

The light charged particles (electrons and positrons) released or produced in the absorbing medium through various photon interactions will either:

(a) Deposit their energy to the medium through Coulomb interactions with orbital electrons of the absorbing medium (collision loss also referred to as ionization loss); or
(b) Radiate their kinetic energy away in the form of photons through Coulomb interactions with the nuclei of the absorbing medium (radiation loss).

Typical examples of the mass attenuation coefficient μ/ρ are shown in Fig. 1.5 with plots of μ/ρ against photon energy $h\nu$ (in solid dark curves) for carbon and lead in the energy range from 0.001 to 1000 MeV. Carbon with $Z = 6$ is an example of a low Z absorber; lead with $Z = 82$ is an example of a high Z absorber. Comparing the two absorbers, it can be noted that at intermediate photon energies (around 1 MeV), carbon and lead have a similar μ/ρ of about 0.1 cm^2/g. On the other hand, at low photon energies, the μ/ρ of lead significantly exceeds the μ/ρ of carbon, and at energies above 10 MeV, the μ/ρ of carbon is essentially flat while the μ/ρ of lead increases with increasing energy.

1.6.4. Photon interactions on the microscopic scale

The general trends in μ/ρ depicted in Fig. 1.5 reflect the elaborate dependence of μ/ρ on the energy $h\nu$ of the photon and the atomic number Z of the absorber. In penetrating an absorbing medium, photons may experience various interactions with the atoms of the medium. On a microscopic scale, these interactions involve either the nuclei of the absorbing medium or the orbital electrons of the absorbing medium:

(a) Photon interactions with the nucleus of the absorber atoms may be direct photon–nucleus interactions (photonuclear reaction) or interactions between the photon and the electrostatic field of the nucleus (nuclear pair production);
(b) Photon interactions with orbital electrons of absorber atoms are characterized as interactions between the photon and either a loosely bound electron (Compton effect, triplet production) or a tightly bound electron (photoelectric effect, Rayleigh scattering).

A loosely bound electron is an electron whose binding energy E_B is much smaller in comparison with the photon energy $h\nu$, i.e. $E_B \ll h\nu$. An interaction between a photon and a loosely bound electron is considered to be an interaction between a photon and a 'free' (i.e. unbound) electron.

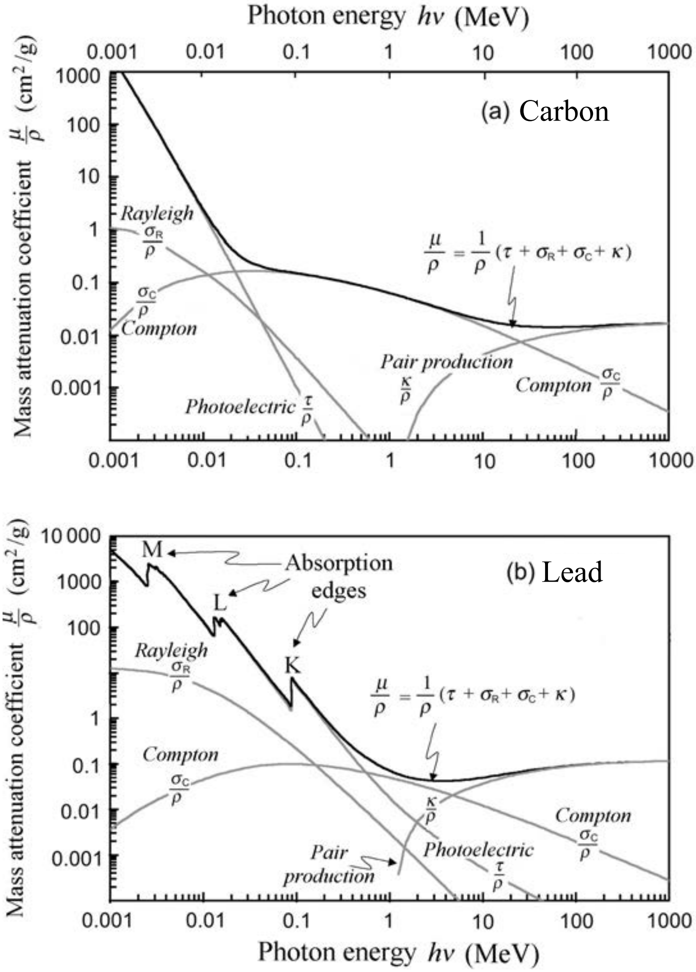

FIG. 1.5. Mass attenuation coefficient μ/ρ against photon energy hν in the range from 1 keV to 1000 MeV for carbon (a) and lead (b). In addition to the total coefficients μ/ρ, the individual coefficients for the photoelectric effect, Rayleigh scattering, Compton scattering and pair production (including triplet production) are also shown. Data are from the National Institute of Science and Technology (NIST).

A tightly bound electron is an electron whose binding energy E_B is comparable to, larger than or slightly smaller than the photon energy $h\nu$. For a photon interaction to occur with a tightly bound electron, the binding energy E_B of the electron must be of the order of, but slightly smaller than, the photon energy, i.e. $E_B \leq h\nu$. An interaction between a photon and a tightly bound electron is considered an interaction between a photon and the atom as a whole.

As far as the photon fate after the interaction with an atom is concerned, there are two possible outcomes: (i) the photon disappears and is absorbed completely (photoelectric effect, nuclear pair production, triplet production, photonuclear reaction) and (ii) the photon is scattered and changes its direction but keeps its energy (Rayleigh scattering) or loses part of its energy (Compton effect).

The most important photon interactions with atoms of the absorber are: the Compton effect, photoelectric effect, nuclear pair production, electronic pair production (triplet production) and photonuclear reactions. In some of these interactions, energetic electrons are released from absorber atoms (photoelectric effect, Compton effect, triplet production) and electronic vacancies are left in absorber atoms; in other interactions, a portion of the incident photon energy is used to produce free electrons and positrons. All of these light charged particles move through the absorber and either deposit their kinetic energy in the absorber (dose) or transform part of it back into radiation through production of bremsstrahlung radiation.

The fate of electronic vacancies produced in photon interactions with absorber atoms is the same as the fate of vacancies produced in electron capture and internal conversion. As alluded to in Section 1.4.11, an electron from a higher atomic shell of the absorber atom fills the electronic vacancy in a lower shell and the transition energy is emitted either in the form of a characteristic X ray (also called a fluorescence photon) or an Auger electron and this process continues until the vacancy migrates to the outer shell of the absorber atom. A free electron from the environment will eventually fill the outer shell vacancy and the absorber ion will revert to a neutral atom in the ground state.

A vacancy produced in an inner shell of an absorber atom migrates to the outer shell and the migration is accompanied by emission of a series of characteristic photons and/or Auger electrons. The phenomenon of emission of Auger electrons from an excited atom is called the Auger effect. Since each Auger transition converts an initial single electron vacancy into two vacancies, a cascade of low energy Auger electrons is emitted from the atom. These low energy electrons have a very short range in tissue but may produce ionization densities comparable to those produced in an α particle track.

The branching between a characteristic photon and an Auger electron is governed by the fluorescence yield ω which, as shown in Fig. 1.6, for a given electronic shell, gives the number of fluorescence photons emitted per vacancy in the shell. The fluorescence yield ω can also be defined as the probability of emission of a fluorescence photon for a given shell vacancy. Consequently, as also shown in Fig. 1.6, $(1 - \omega)$ gives the probability of emission of an Auger electron for a given shell vacancy.

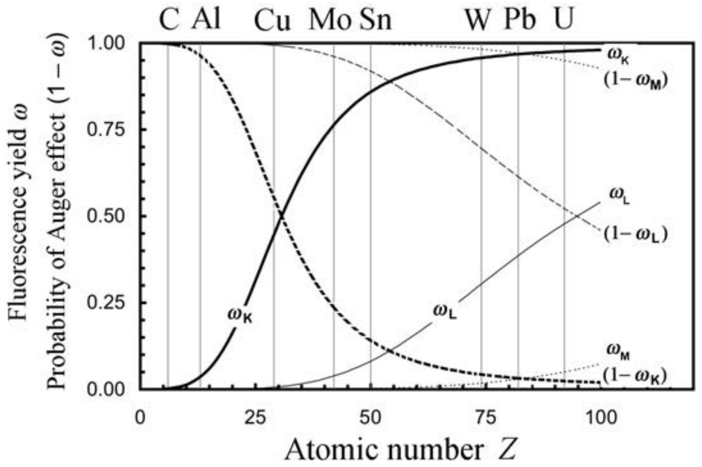

FIG. 1.6. Fluorescence yields ω_K*,* ω_L *and* ω_M*, against atomic number Z of the absorber. Also shown are probabilities for the Auger effect, given as (1 – ω). Data are from the National Institute of Science and Technology (NIST).*

1.6.5. Photoelectric effect

In the photoelectric effect (sometimes called the ‘photoeffect’), the photon interacts with a tightly bound orbital electron of an absorber atom, the photon disappears and the orbital electron is ejected from the atom as a so-called photoelectron, with a kinetic energy E_K given as:

$$E_K = h\nu - E_B \tag{1.56}$$

where

$h\nu$ is the incident photon energy;

and E_B is the binding energy of the ejected photoelectron.

A general diagram of the photoelectric effect is provided (see Fig. 1.9(a)).

For the photoelectric effect to happen, the photon energy $h\nu$ must exceed the binding energy E_B of the orbital electron to be ejected and, moreover, the closer $h\nu$ is to E_B, the higher the probability of the photoelectric effect happening. The photoelectric mass attenuation coefficient τ/ρ is plotted in Fig. 1.5 for carbon and lead as one of the grey curves representing the components of the total μ/ρ attenuation coefficient. The sharp discontinuities in the energy $h\nu$ are called absorption edges and occur when $h\nu$ becomes equal to the binding energy E_B of a given atomic shell. For example, the K absorption edge occurs at $h\nu = 88$ keV in lead, since the K shell binding energy E_B in lead is 88 keV. Absorption edges for carbon occur at $h\nu < 1$ keV and, thus, do not appear in Fig. 1.5(a).

As far as the photoelectric attenuation coefficient dependence on photon energy $h\nu$ and absorber atomic number Z is concerned, the photoelectric atomic attenuation coefficient ${}_a\tau$ goes approximately as $Z^5/(h\nu)^3$, while the photoelectric mass attenuation coefficient τ/ρ goes approximately as $Z^4/(h\nu)^3$.

As evident from Fig. 1.5, the photoelectric attenuation coefficient τ/ρ is the major contributor to the total attenuation coefficient μ/ρ at relatively low photon energies where $h\nu$ is of the order of the K shell binding energy and less than 0.1 MeV. At higher photon energies, first the Compton effect and then pair production become the major contributors to the photon attenuation in the absorber.

1.6.6. Rayleigh (coherent) scattering

In Rayleigh scattering (also called ‘coherent scattering’), the photon interacts with the full complement of tightly bound atomic orbital electrons of an absorber atom. The event is considered elastic in the sense that the photon loses essentially none of its energy $h\nu$ but is scattered through a relatively small scattering angle θ. A general diagram of Rayleigh scattering is given (see Fig. 1.9(b)).

Since no energy transfer occurs from photons to charged particles, Rayleigh scattering plays no role in the energy transfer attenuation coefficient and energy absorption coefficient; however, it contributes to the total attenuation coefficient μ/ρ through the elastic scattering process. The Rayleigh atomic attenuation coefficient ${}_a\sigma_R$ is proportional to $Z^2/(h\nu)^2$ and the Rayleigh mass attenuation coefficient σ_R/ρ is proportional to $Z/(h\nu)^2$.

As a result of no energy transfer from photons to charged particles in the absorber, Rayleigh scattering is of no importance in radiation dosimetry. As far as photon attenuation is concerned, however, the relative importance of Rayleigh scattering in comparison to other photon interactions in tissue and tissue equivalent materials amounts to only a few per cent of the total μ/ρ but it should not be neglected.

1.6.7. Compton effect (incoherent scattering)

The Compton effect (also called ‘incoherent scattering’ or ‘Compton scattering’) is described as an interaction between a photon and a free as well as stationary electron. Of course, the interacting electron is not free, rather it is bound to a nucleus of an absorbing atom, but the photon energy $h\nu$ is much larger than the binding energy E_B of the electron ($E_B \ll h\nu$), so that the electron is said to be loosely bound or essentially ‘free and stationary’.

In the Compton effect, the photon loses part of its energy to the recoil (Compton) electron and is scattered as a photon $h\nu'$ through a scattering angle θ, as shown schematically in Fig. 1.7. In the diagram, the interacting electron is at the origin of the Cartesian coordinate system and the incident photon is oriented in the positive direction along the abscissa (x) axis. The scattering angle θ is the angle between the direction of the scattered photon $h\nu'$ and the positive abscissa axis while the recoil angle ϕ is the angle between the direction of the recoil electron and the positive abscissa axis. A general diagram of the Compton effect is given (see Fig. 1.9(c)).

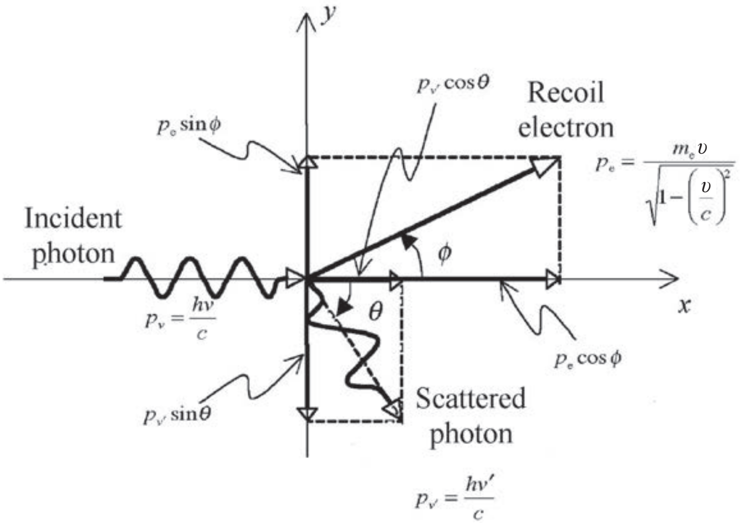

FIG. 1.7. Schematic diagram of the Compton effect in which an incident photon of energy $h\nu$ = 1 MeV interacts with a ‘free and stationary’ electron. A photon with energy $h\nu'$ = 0.505 MeV is produced and scattered with a scattering angle θ = 60°.

Considerations of conservation of energy and momentum result in the following three equations for the Compton effect:

(a) Conservation of energy:

$$h\nu + m_e c^2 = h\nu' + m_e c^2 + E_K \qquad \text{or} \qquad h\nu = h\nu' + E_K \tag{1.57}$$

(b) Conservation of momentum on the abscissa (x) axis:

$$p_\nu = \frac{h\nu'}{c}\cos\theta + \frac{m_e \upsilon}{\sqrt{1-\frac{\upsilon^2}{c^2}}}\cos\phi \tag{1.58}$$

(c) Conservation of momentum on the ordinate (y) axis:

$$0 = -\frac{h\nu'}{c}\sin\theta + \frac{m_e \upsilon}{\sqrt{1-\frac{\upsilon^2}{c^2}}}\sin\phi \tag{1.59}$$

where

$m_e c^2$ is the rest energy of the electron (0.511 MeV);
E_K is the kinetic energy of the recoil (Compton) electron;
υ is the velocity of the recoil (Compton) electron;

and c is the speed of light in a vacuum (3×10^8 m/s).

From equations describing conservation of energy (Eq. (1.57)) and conservation of momentum (Eqs (1.58) and (1.59)), the basic Compton equation (also referred to as the Compton wavelength-shift equation) can be derived and is expressed as follows:

$$\lambda' - \lambda = \Delta\lambda = \frac{h}{m_e c}(1-\cos\theta) = \lambda_C(1-\cos\theta) \tag{1.60}$$

where

λ is the wavelength of the incident photon (c/ν);
λ' is the wavelength of the scattered photon (c/ν');
$\Delta\lambda$ is the wavelength shift in Compton effect ($\lambda' - \lambda$);

and λ_C, defined as $\lambda_C = h/(m_e c) = 2\pi\hbar c/(m_e c^2) = 0.024$ Å, is the so-called Compton wavelength of the electron.

From the Compton equation (Eq. (1.60)), it is easy to show that the scattered photon energy $h\nu'$ and the recoil electron kinetic energy E_K depend on the incident photon energy $h\nu$ as well as on the scattering angle θ and are, respectively, given as:

$$h\nu'(h\nu,\theta) = h\nu \frac{1}{1+\varepsilon(1-\cos\theta)} \tag{1.61}$$

and

$$E_{\mathrm{K}}^{\mathrm{C}}(h\nu,\theta) = h\nu - h\nu' = h\nu - h\nu \frac{1}{1+\varepsilon(1-\cos\theta)} = h\nu \frac{\varepsilon(1-\cos\theta)}{1+\varepsilon(1-\cos\theta)} \tag{1.62}$$

where ε is the incident photon energy $h\nu$ normalized to electron rest energy $m_{\mathrm{e}}c^2$, i.e. $\varepsilon = h\nu/(m_{\mathrm{e}}c^2)$.

Using Eq. (1.61), it is easy to show that energies of forward-scattered photons ($\theta = 0$), side-scattered photons ($\theta = \pi/2$) and backscattered photons ($\theta = \pi$) are in general given as follows:

$$h\nu'\big|_{\theta=0} = h\nu \tag{1.63}$$

$$h\nu'\big|_{\theta=\frac{\pi}{2}} = \frac{h\nu}{1+\varepsilon} \tag{1.64}$$

and

$$h\nu'\big|_{\theta=\pi} = \frac{h\nu}{1+2\varepsilon} \tag{1.65}$$

For very large incident photon energies ($h\nu \to \infty$), they are given as:

$$h\nu'\big|_{\theta=0} = h\nu \tag{1.66}$$

$$h\nu'\big|_{\theta=\frac{\pi}{2}} = m_{\mathrm{e}}c^2 \tag{1.67}$$

and

$$h\nu'\big|_{\theta=\pi} = \frac{m_{\mathrm{e}}c^2}{2} \tag{1.68}$$

From the conservation of momentum equations (Eqs (1.58) and (1.59)), the following expression for the relationship between the scattering angle θ and recoil electron angle ϕ can be derived:

$$\cot\phi = (1+\varepsilon)\tan\frac{\theta}{2} \tag{1.69}$$

and

$$\tan\phi = \frac{1}{1+\varepsilon}\cot\frac{\theta}{2} \tag{1.70}$$

Since the range of θ is from 0 (forward-scattering) through $\pi/2$ (side-scattering) to π (backscattering), it is noted that the corresponding range of ϕ is from $\phi = \pi/2$ at $\theta = 0$ through to $\phi = (1 + \varepsilon)^{-1}$ for $\theta = \pi/2$ to $\phi = 0$ at $\theta = \pi$.

The Compton electronic attenuation coefficient ${}_{\mathrm{e}}\sigma_{\mathrm{C}}$ steadily decreases with increasing $h\nu$ from a theoretical value of 0.665×10^{-24} cm^2/electron (known as the Thomson cross-section) at low photon energies to 0.21×10^{-24} cm^2/electron at $h\nu = 1$ MeV, 0.51×10^{-24} cm^2/electron at $h\nu = 10$ MeV, and 0.008×10^{-24} cm^2/electron at $h\nu = 100$ MeV.

Since Compton interaction is a photon interaction with a free electron, the Compton atomic attenuation coefficient ${}_{\mathrm{a}}\sigma_{\mathrm{C}}$ depends linearly on the absorber atomic number Z, while the electronic coefficient ${}_{\mathrm{e}}\sigma_{\mathrm{C}}$ and the mass coefficient σ_{C}/ρ are essentially independent of Z. This independence of Z can be observed in Fig. 1.5, showing that σ_{C}/ρ for carbon ($Z = 6$) and lead ($Z = 82$) at intermediate photon energies (~1 MeV), where Compton effect predominates, are equal to about 0.1 cm^2/electron irrespective of Z.

Equation (1.62) gives the energy transferred from the incident photon to the recoil electron in the Compton effect as a function of the scattering angle θ. The maximum energy transfer to recoil electron occurs when the photon is backscattered ($\theta = \pi$) and the Compton maximum energy transfer fraction $(f_{\mathrm{C}})_{\max}$ is then given as:

$$(f_{\mathrm{C}})_{\max} = \frac{(E_{\mathrm{K}}^{\mathrm{C}})_{\max}}{h\nu} = \frac{2\varepsilon}{1+2\varepsilon} \tag{1.71}$$

The mean energy transfer in the Compton effect $\bar{f}_{\mathrm{C}}$ is determined by normalizing (to incident photon energy $h\nu$) the mean energy transferred to the Compton electron $\bar{E}_{\mathrm{K}}^{\mathrm{C}}$. This quantity is very important in radiation dosimetry and is plotted against incident photon energy $h\nu$ in the Compton graph presented in Fig. 1.8. The figure shows that the fractional energy transfer to recoil electrons

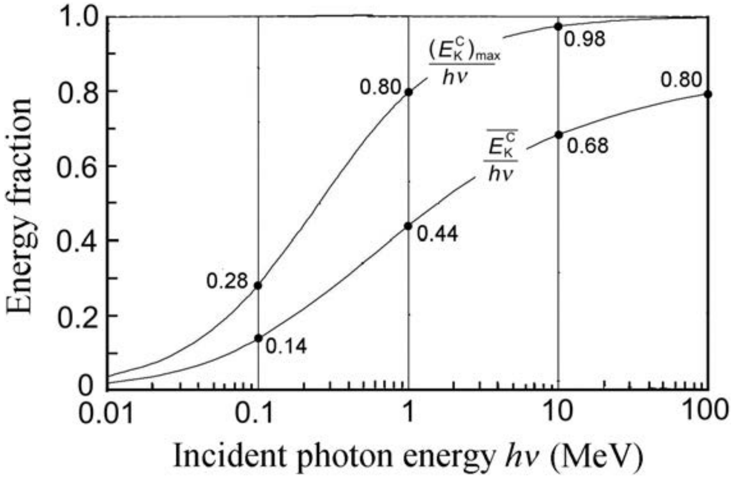

FIG. 1.8. Maximum and mean fractions of incident photon energy hv transferred to the recoil electron in the Compton effect. Data are from the National Institute of Science and Technology (NIST).

is quite low at low photon energies ($\overline{f}_C = 0.02$ at $h\nu = 0.01$ MeV) and then slowly rises through $\overline{f}_C = 0.44$ at $h\nu = 1$ MeV to reach $\overline{f}_C = 0.80$ at $h\nu = 100$ MeV and approaches one asymptotically at very high incident photon energies.

1.6.8. Pair production

When the incident photon energy $h\nu$ exceeds $2m_ec^2 = 1.022$ MeV, with m_ec^2 being the rest energy of the electron and positron, the production of an electron–positron pair in conjunction with a complete absorption of the incident photon by the absorber atom becomes energetically possible. For the effect to occur, three quantities must be conserved: energy, charge and momentum. To conserve the linear momentum simultaneously with total energy and charge, the effect cannot occur in free space; it can only occur in the Coulomb electric field of a collision partner (atomic nucleus or orbital electron) that can take up a suitable fraction of the momentum carried by the photon. Two types of pair production are known:

— If the collision partner is an atomic nucleus of the absorber, the pair production event is called nuclear pair production and is characterized by a photon energy threshold slightly larger than two electron rest masses ($2m_ec^2 = 1.022$ MeV).

— Less probable, but nonetheless possible, is pair production in the Coulomb field of an orbital electron of an absorber atom. The event is called electronic pair production or triplet production and its threshold photon energy is $4m_ec^2 = 2.044$ MeV.

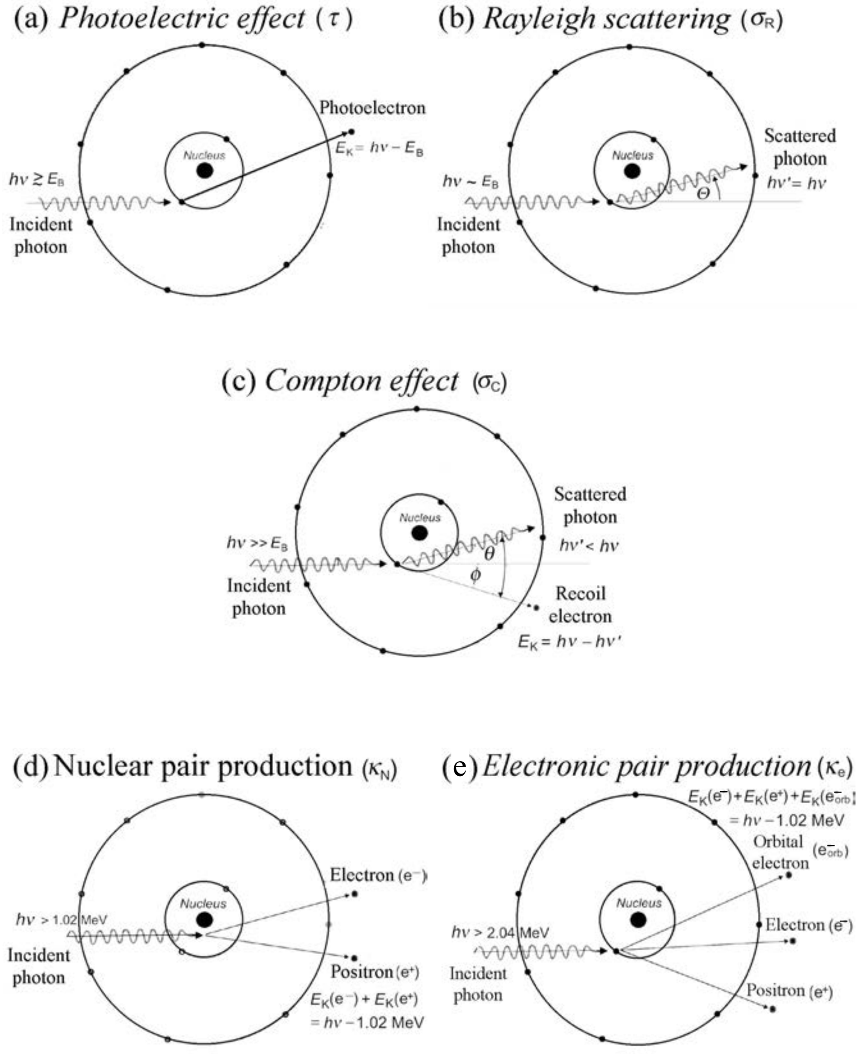

FIG. 1.9. Schematic diagrams of the most important modes of photon interaction with atoms of an absorber: (a) photoelectric effect; (b) Rayleigh scattering; (c) Compton effect; (d) nuclear pair production; and (e) electronic pair production (triplet production).

The two pair production attenuation coefficients, despite having different origins, are usually dealt with together as one parameter referred to as pair production. The component that the nuclear pair production contributes usually exceeds 90%. Nuclear pair production and electronic pair/triplet production are shown schematically in Figs 1.9(d) and (e), respectively.

The probability of pair production is zero for photon energy below the threshold value and increases rapidly with photon energy above the threshold. The pair production atomic attenuation coefficient ${}_a\kappa$ and the pair production mass attenuation coefficient κ/ρ vary approximately as Z^2 and Z, respectively, where Z is the atomic number of the absorber.

1.6.9. Relative predominance of individual effects

As is evident from the discussion above, photons have several options for interaction with absorber atoms. Five of the most important photon interactions are shown schematically in Fig. 1.9. Nuclear and electronic pair production are usually combined and treated under the header ‘pair production’.

The probability for a photon to undergo any one of the various interaction phenomena with an absorber depends on the energy $h\nu$ of the photon and the atomic number Z of the absorber. In general, the photoelectric effect predominates at low photon energies, the Compton effect at intermediate energies and pair production at high photon energies.

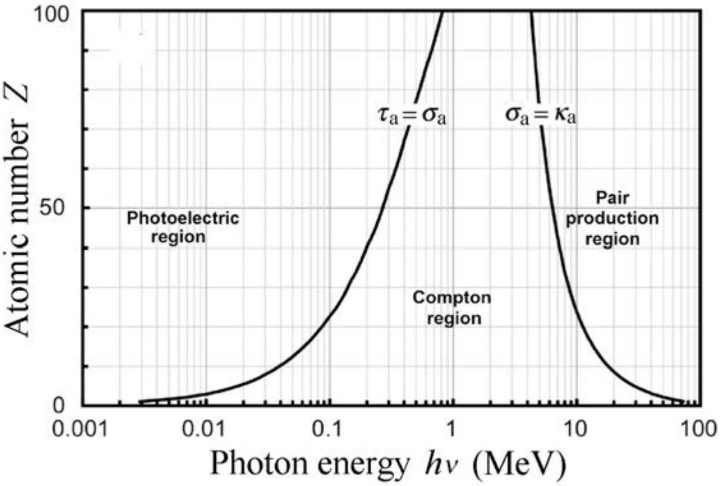

FIG. 1.10. Representation of the relative predominance of the three main processes of photon interaction with an absorber atom: the photoelectric effect, Compton effect and pair production.

Figure 1.10 shows the regions of relative predominance of the three most important individual effects with *hv* and *Z* as parameters. The two curves display the points on the (*hv*, *Z*) diagram for which $\sigma_C = \tau$ at low photon energies and for which $\sigma_C = \kappa$ for high photon energies and, thus, delineate regions of photoelectric effect predominance at low photon energies, Compton effect predominance at intermediate photon energies and pair production predominance at high photon energies. Figure 1.10 also indicates how the regions of predominance are affected by the absorber atomic number. For example, a 100 keV photon will interact with a lead absorber ($Z = 82$) predominantly through the photoelectric effect and with soft tissue ($Z_{\text{eff}} \approx 7.5$) predominantly through the Compton effect. A 10 MeV photon, on the other hand, will interact with lead predominantly through pair production and with tissue predominantly through the Compton effect.

1.6.10. Macroscopic attenuation coefficients

For a given photon energy *hv* and absorber atomic number *Z*, the macroscopic attenuation coefficient μ and energy transfer coefficient μ_{tr} are given as a sum of coefficients for individual photon interactions discussed above (photoelectric, Rayleigh, Compton and pair production):

$$\mu = \rho \frac{N_{\text{A}}}{A} ({}_{\text{a}}\tau + {}_{\text{a}}\sigma_{\text{R}} + {}_{\text{a}}\sigma_{\text{C}} + {}_{\text{a}}\kappa) \tag{1.72}$$

and

$$\mu_{\text{tr}} = \rho \frac{N_{\text{A}}}{A} \left[{}_{\text{a}}\tau_{\text{tr}} + ({}_{\text{a}}\sigma_{\text{C}})_{\text{tr}} + {}_{\text{a}}\kappa_{\text{tr}} \right] = \rho \frac{N_{\text{A}}}{A} \left[{}_{\text{a}}\tau\, \bar{f}_{\text{PE}} + {}_{\text{a}}\sigma_{\text{C}}\, \bar{f}_{\text{C}} + {}_{\text{a}}\kappa\, \bar{f}_{\text{PP}} \right] \tag{1.73}$$

where all parameters are defined in sections dealing with the individual microscopic effects.

It should be noted that in Rayleigh scattering there is no energy transfer to charged particles.

The energy absorption coefficient μ_{ab} (often designated μ_{en} in the literature) is derived from μ_{tr} of Eq. (1.73) as follows:

$$\mu_{\text{ab}} = \mu_{\text{en}} = \mu_{\text{tr}} (1 - \bar{g}) \tag{1.74}$$

where $\bar{g}$ is the mean radiation fraction accounting for the fraction of the mean energy transferred from photons to charged particles and subsequently lost by charged particles through radiation losses. These losses consist of two components: the predominant bremsstrahlung loss and the small, yet not always negligible, in-flight annihilation loss.

1.6.11. Effects following photon interactions with absorber and summary of photon interactions

In the photoelectric effect, Compton effect and triplet production, vacancies are produced in atomic shells of absorber atoms through the ejection of orbital electrons from atomic shells. For the diagnostic range and megavoltage range of photons used for diagnosis and treatment of disease with radiation, the shell vacancies occur mainly in inner atomic shells and are followed by characteristic radiation or Auger electrons, the probability of the former given by fluorescence yield ω (see Fig. 1.6).

Pair production and triplet production are followed by the annihilation of the positron with a 'free' electron producing two annihilation quanta, most commonly with an energy of 0.511 MeV each and emitted at 180º from each other to satisfy conservation of energy, momentum and charge.

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