

CHAPTER 5

STATISTICS FOR RADIATION MEASUREMENT

M.G. LÖTTER
Department of Medical Physics,
University of the Free State,
Bloemfontein, South Africa

5.1. SOURCES OF ERROR IN NUCLEAR MEDICINE MEASUREMENT

Measurement errors are of three general types: (i) blunders, (ii) systematic errors or accuracy of measurements, and (iii) random errors or precision of measurements.

Blunders produce grossly inaccurate results and experienced observers easily detect their occurrence. Examples in radiation counting or measurements include the incorrect setting of the energy window, counting heavily contaminated samples, using contaminated detectors for imaging or counting, obtaining measurements of high activities, resulting in count rates that lead to excessive dead time effects, and selecting the wrong patient orientation during imaging. Although some blunders can be detected as outliers or by duplicate samples and measurements, blunders should be avoided by careful, meticulous and dedicated work. This is especially important where results will determine the diagnosis or treatment of patients.

Systematic errors produce results that differ consistently from the correct results by some fixed amount. The same result may be obtained in repeated measurements, but overestimating or underestimating the true value. Systematic errors are said to influence the accuracy of measurements. Measurement results having systematic errors will be inaccurate or biased. Examples of a systematic error are:

- When an incorrectly calibrated ionization chamber is used for measurement of radiation dose.
- When during thyroid uptake studies with ^{123}I the count rate of the reference standard results in dead time losses. The percentage of thyroid uptake will be overestimated.
- When in sample counting the geometry of samples and the position within the detector are not the same as in the reference sample.

- When during blood volume measurements the tracer leaks out of the blood compartment. The theory of the method assumes that the tracer will stay in the blood compartment. The leaking of the tracer will consistently overestimate the measured blood volume.
- When in calculation of the ventricular ejection fraction during gated blood pool studies the selected background counts underestimate the true ventricular background counts, the ejection fraction will be consistently underestimated.

Measurement results affected by systematic errors are not always easy to detect, since the measurements may not be too different from the expected results. Systematic errors can be detected by using reference standards. For example, radionuclide standards calibrated at a reference laboratory should be used to calibrate source calibrators to determine correction factors for each radionuclide used for patient treatment and diagnosis.

Measurement results affected by systematic errors can differ from the true value by a constant value and/or by a fraction. Using 'golden standard' reference values, a regression curve can be calculated. The regression curve can be used to convert systematic errors to a more accurate value. For example, if the ejection fraction is determined by a radionuclide gated study, it can be correlated with the 'golden standard' values.

Random errors are variations in results from one measurement to the next, arising from actual random variation of the measured quantity itself, as well as physical limitations of the measurement system.

Random error affects the reproducibility, precision or uncertainty in the measurement. Random errors are always present when radiation measurements are performed because the measured quantity, namely the radionuclide decay, is a random varying quantity. The random error during radiation measurements introduced by the measured quantity, that is the radionuclide decay, is demonstrated in Fig. 5.1. Figure 5.1 shows the energy spectrum of a ^{57}Co source in a scattering medium and measured with a scintillation detector probe. The energy spectrum represented by square markers is the measured energy spectrum with random noise due to radionuclide decay. The solid line spectrum represents the energy spectrum without random noise. The variation around the solid line of the data points, represented by markers, is a result of random error introduced by radionuclide decay.

The influence of the random error of the measurement system introduced by the scintillation detector is also demonstrated in Fig. 5.1. Cobalt-57 emits photons of 122 keV and with a perfect detection system all of the counts are expected at 122 keV. The measurements are, however, spread around 122 keV as a result of the random error introduced by the scintillation detector during the

detection of each γ photon. When a γ photon is detected with the scintillation detector, the number of charge carriers generated will vary randomly. The varying number of charge carriers will cause varying pulse heights at the output of the detector and this variation determines the spread around the true photon energy of 122 keV. The width of the photopeak determines the energy resolution of the detection system.

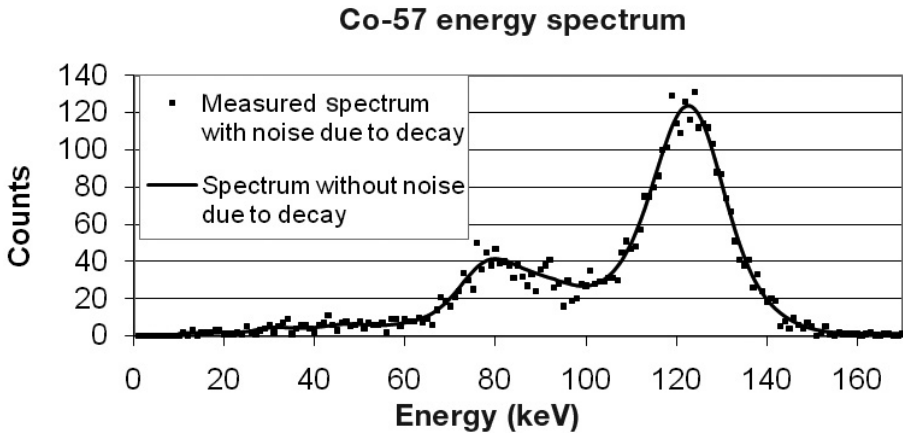


FIG. 5.1. Energy spectrum of a ^{57}Co source in a scattering medium obtained with a scintillation detector.

Random errors also play a significant role in radionuclide imaging. Here, the random error as a result of the measured quantity, namely radionuclide decay, will significantly influence the visual quality of the image. This is because the number of counts acquired in each pixel is subject to random error. It is shown that the relative random error decreases as the number of counts per pixel increases. The visual effect of the random error as a result of the measured quantity is demonstrated in Fig. 5.2. Technetium-99m planar bone scans (acquired on a 256×256 matrix) were acquired with a scintillation camera. Image acquisition was terminated at a total count of 21, 87 and 748 kcounts. When the total number of counts per image are increased, the counts per pixel increase and the random error decreases, resulting in improved visual image quality. As the accumulated counts are increased, the ability to visualize anatomical structures and, more importantly, tumour volumes, significantly increases. The random error introduced by the measuring system or imaging device, such as a scintillation camera, also influences image quality. This is as a result of the energy resolution and intrinsic spatial resolution of imaging devices that are influenced by random errors during the detection of each γ photon. The energy resolution of the system

will determine the ability of the system to reject lower energy scattered γ photons and improve image contrast.

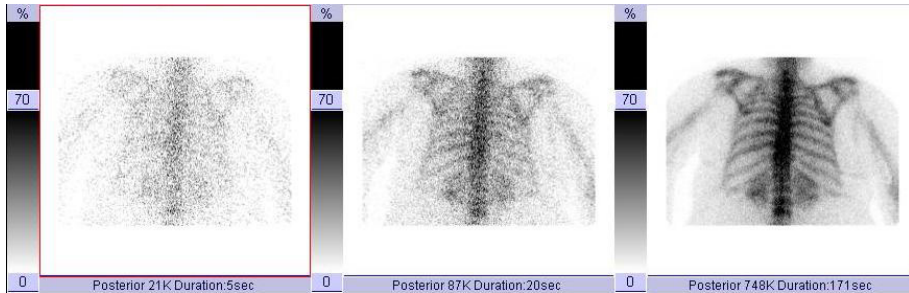


FIG. 5.2. The influence of random error as a result of radionuclide decay or counting statistics is demonstrated for imaging. Technetium-99m posterior planar bone images (256×256) using a scintillation camera were acquired to total counts of 21, 87 and 748 kcounts.

It is possible for a measurement to be precise (small random error) but inaccurate (large systematic error), or vice versa. For example, for the calculation of the ejection fraction during gated cardiac studies, the selection of the background region of interest (ROI) will be exactly reproducible when a software algorithm is used. However, if the algorithm is such that the selected ROI does not reflect the true ventricular background, the measurement will be precise but inaccurate. Conversely, individual radiation counts of a radioactive sample may be imprecise because of the random error, but the average value of a number of measurements will be accurate, representing the true counts acquired.

Random errors are always present and play a significant role in radiation counting and imaging. It is, therefore, important to analyse the random errors to determine the associated uncertainty. This is done using methods of statistical analysis. The remainder of the chapter describes methods of analysis.

The analysis of radiation measurements and imaging forms a subgroup of general statistical analysis. In this chapter, the focus is on statistical analysis for radiation counting and imaging measurements, although some methods described will be applicable to a wider class of experimental data as described in Sections 5.2, 5.3 and 5.5.

5.2. CHARACTERIZATION OF DATA

5.2.1. Measures of central tendency and variability

5.2.1.1. Dataset as a list

Two measurements of the central tendency of a set of measurements are the mean (average) and median. It is assumed that there is a list of N independent measurements of the same physical quantity:

$$x_1, x_2, x_3, \dots, x_i, \dots, x_N$$

It is supposed that the dataset is obtained from a long lived radioactive sample counted repeatedly under the same conditions with a properly operating counting system. As the disintegration rate of the radioactive sample undergoes random variations from one moment to the next, the number of counts recorded in successive measurements is not the same as the result of random errors in the measurement.

The experimental mean \bar{x}_e of the set of measurements is defined as:

$$\bar{x}_e = \frac{x_1 + x_2 + \dots + x_N}{N} \tag{5.1}$$

$$= \frac{\sum_{i=1}^N x_i}{N} \tag{5.2}$$

The following procedure is followed to obtain the median. The list of measurements must first be sorted by size. The median is the middlemost measurement if the number of measurements is odd and is the average of the two middlemost measurements if the number of measurements is even. For example, to obtain the median of five measurements, 7, 13, 6, 10 and 14, they are first sorted by size: 6, 7, 10, 13 and 14. The median is 10. The advantage of the median over the mean is that the median is less affected by outliers. An outlier is a blunder and is much greater or much less than the others.

The measures of variability, random error and precision of a list of measurements are the variance, standard deviation and fractional standard deviation, respectively.

The variance σ_e^2 is determined from a set of measurements by subtracting the mean from each measurement, squaring the difference, summing the squares and dividing by one less than the number of measurements:

$$\begin{aligned}\sigma_e^2 &= \frac{(x_1 - \bar{x}_e)^2 + (x_2 - \bar{x}_e)^2 + \dots + (x_N - \bar{x}_e)^2}{N - 1} \\ &= \frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{x}_e)^2\end{aligned}\quad (5.3)$$

where N is the total number of measurements and \bar{x}_e is the experimental mean.

The standard deviation σ_e is the square root of the variance:

$$\sigma_e = \sqrt{\sigma_e^2} \quad (5.4)$$

The fractional standard deviation σ_{eF} (fractional error or coefficient of variation) is the standard deviation divided by the mean:

$$\sigma_{eF} = \frac{\sigma_e}{\bar{x}_e} \quad (5.5)$$

The fractional standard deviation is an important measure to evaluate variability in measurements of radioactivity. The inverse of the fractional standard deviation $1/\sigma_{eF}$ in imaging is referred to as the signal to noise ratio.

5.2.1.2. Dataset as a relative frequency distribution function

It is often convenient to represent the dataset by a relative frequency distribution function $F(x)$. The value of $F(x)$ is the relative frequency with which the number appears in the collection of data in each bin. By definition:

$$F(x) = \frac{\text{number of occurrences of the value } x \text{ in each bin}}{\text{number of measurements } (N)} \quad (5.6)$$

The distribution is normalized, that is:

$$\sum_{x=0}^{\infty} F(x) = 1 \quad (5.7)$$

As long as the specific sequence of numbers is not important, the complete data distribution function represents all of the information in the original dataset in list format.

Figure 5.3 illustrates a demonstration of the application of the relative frequency distribution. The scintillation counter measurements appear noisy due to the random error as a result of the measured quantity (the radionuclide decay) (Fig. 5.3(a)). The measurements fluctuate randomly above and below the mean of 90 counts. A histogram (red bars) of the relative frequency distribution of the fluctuations in the measurements can be constructed by plotting the relative frequency of the measured counts (Fig. 5.3(b)). The x axis represents the range of possible counts that were measured in each of the bins with a six count interval. The y axis represents the relative frequencies with which the particular count values occur. The most common value, that is 26% of the measurements, is near the mean of 90 counts. The values of the other measurements are substantially higher or lower than the mean. The measured frequency distribution histogram agrees well with the expected calculated normal distribution (blue curve).

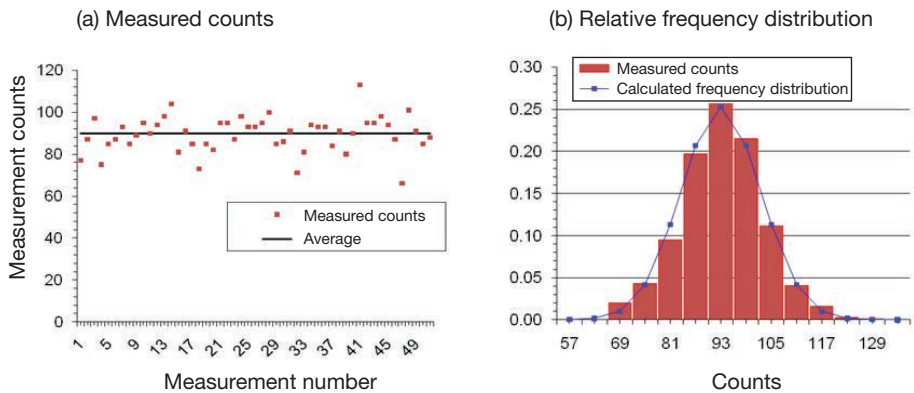


FIG. 5.3. One thousand measurements were made with a scintillation counter. (a) The graph shows the variations observed for the first 50 measurements. (b) The graph (red bars) shows the histogram of the relative frequency distribution for the measurements as well as the expected calculated frequency distribution.

The relative frequency distribution is a useful tool to provide a quick visual summary of the distribution of measurement values and can be used to identify outliers such as blunders or the correct functioning of equipment.

Three measurements of the central tendency for a frequency distribution are the mean (average), median and mode:

- The mode of a frequency distribution is defined as the most frequent value or the value at the maximum probability of the frequency distribution.
- The median of a frequency distribution is the value at which the integral of the frequency distribution is 0.5; that is, half of the measurements will be smaller and half will be larger than the median.

The experimental mean \bar{x}_e using the frequency distribution function can be calculated. The experimental mean is obtained by calculating the first moment of the frequency distribution function. The equation for calculating the mean can also be derived from the equation for calculating the mean for data in a list (Eq. (5.2)). The sum of measurements $\sum_{i=1}^N x_i$ is equal to the sum of the measurements in each ‘bin’ in the frequency distribution function. The sum of the measurements for each bin is obtained by multiplying the value of the bin i and the number of occurrences of the value x_i .

$$\begin{aligned}\bar{x}_e &= \frac{\sum_{i=1}^N x_i}{N} \\ &= \frac{\sum_{i=1}^N [\text{value of bin } (i)] [\text{number of occurrences of the value } x_i]}{N} \\ &= \sum_{i=1}^N x_i F(x_i)\end{aligned}\quad (5.8)$$

The experimental sample variance can be calculated using the frequency distribution function:

$$\sigma_e^2 = \frac{N}{N-1} \sum_{i=1}^N (x_i - \bar{x}_e)^2 F(x_i) \quad (5.9)$$

The standard deviation and the fractional standard deviation are given by Eqs (5.4) and (5.5).

The frequency distribution provides information and insight on the precision of the experimental sample mean and of a single measurement. Figure 5.3 demonstrates the distribution of counting measurements around the

true mean (\bar{x}_t). The value of the true mean is not known but the experimental sample mean (\bar{x}_e) can be used as an estimate of the true mean (\bar{x}_t):

$$(\bar{x}_t) \approx (\bar{x}_e) \tag{5.10}$$

In routine practice, it is often impractical to obtain multiple measurements and one must be satisfied with only one measurement. This is especially the case during radionuclide imaging and nuclear measurements on patients. The frequency distribution of the measurements will determine the precision of a single measurement as an estimate of the true value. The probability that a single measurement will be close to the true mean depends on the relative width or dispersion of the frequency distribution curve. This is expressed by the variance σ^2 (Eq. (5.9)) or standard deviation σ of the distribution. The standard deviation σ is a number such that 68.3% of the measurement results fall within $\pm\sigma$ of the true mean \bar{x}_t .

Given the result of a given measurement x , it can be said that there is a 68.3% chance that the measurement is within the range $x \pm \sigma$. This is called the 68.3% confidence interval for the true mean \bar{x}_t . There is 68.3% confidence that \bar{x}_t is in the range $x \pm \sigma$. Other confidence intervals can be defined in terms of the standard deviation σ . They are summarized in Table 5.1. The 50% confidence interval (0.675σ) is referred to as the probable error of the true mean \bar{x}_t .

TABLE 5.1. CONFIDENCE LEVELS IN RADIATION MEASUREMENTS

Range	Confidence level for true mean \bar{x}_t
$\bar{x}_t \pm 0.675\sigma$	50.0
$\bar{x}_t \pm 1.000\sigma$	68.3
$\bar{x}_t \pm 1.640\sigma$	90.0
$\bar{x}_t \pm 2.000\sigma$	95.0
$\bar{x}_t \pm 3.000\sigma$	99.7

5.3. STATISTICAL MODELS

Under certain conditions, the distribution function that will describe the results of many repetitions of a given measurement can be predicted. A measurement is defined as counting the number of successes x resulting from a given number of trials n . Each trial is assumed to be a binary process in that only

two results are possible: the trial is either a success or not. It is further assumed that the probability of success p is constant for all trials.

To show how these conditions apply in real situations, Table 5.2 gives four separate examples. The third example gives the basis for counting nuclear radiation events. In this case, a trial consists of observing a given radioactive nucleus for a period of time t . The number of trials n is equivalent to the number of nuclei in the sample under observation, and the measurement consists of counting those nuclei that undergo decay. We identify the probability of success as p . For radioactive decay:

$$p = (1 - e^{-\lambda t}) \quad (5.11)$$

where λ is the decay constant of the radionuclide.

The fifth example demonstrates the uncertainty associated with the energy determination during scintillation counting. The light photons generated in the scintillator following interaction with an incoming γ ray will eject electrons at the photocathode of the photomultiplier tube (PMT). Typically, one electron ejected for every five light photons results in a probability of success of 1/5.

TABLE 5.2. EXAMPLES OF BINARY PROCESSES

Trial	Definition of success	Probability of success p
Tossing a coin	Heads	1/2
Rolling a die	A six	1/6
Observing a given radionuclide for time t	The nucleus decays during observation	$(1 - e^{-\lambda t})$
Observing a given γ ray over a distance x in an attenuating medium	The γ ray interacts with the medium during observation	$(1 - e^{-\mu x})$
Observing light photons generated in a scintillator	An electron is ejected from the photocathode	1/5

5.3.1. Conditions when binomial, Poisson and normal distributions are applicable

Three statistical models are used: the binomial distribution, the Poisson distribution and the Gaussian or normal distribution. Figure 5.4 shows the

distribution for the three models. The distributions were generated by using a Microsoft Office Excel spreadsheet.

5.3.1.1. Binomial distribution

This is the most general model and is widely applicable to all constant p processes (Fig. 5.4). Binomial distribution is rarely used in nuclear decay applications. One example in which the binomial distribution must be used is when a radionuclide with a very short half-life is counted with a high counting efficiency.

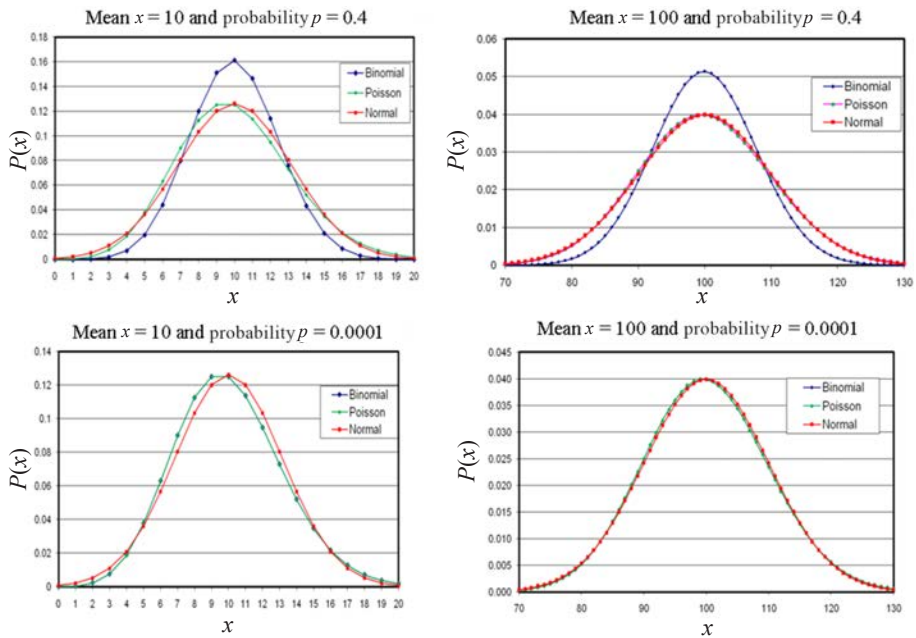


FIG. 5.4. Probability distribution models for successful event probability $p = 0.4$ and $p = 0.0001$ for $\bar{x} = 10$ and $\bar{x} = 100$, respectively.

5.3.1.2. Poisson distribution

The model is a direct mathematical simplification of the binomial distribution under conditions that the event probability of success p is small (Fig. 5.4). For nuclear counting, this condition implies that the chosen observation time is small compared to the half-life of the source, or that the detection

efficiency is low. The Poisson distribution is an important distribution. When the success rate is low, the true experimental distribution is asymmetric and a Poisson distribution must then be used since the normal distribution is always symmetrical. This is demonstrated in Fig. 5.4 for $p = 0.0001$ and $\bar{x} = 10$.

5.3.1.3. *Gaussian or normal distribution*

The third important distribution is the normal or Gaussian, which is a further simplification if the mean number of successes \bar{x} is relatively large (>30). At this level of success, the experimental distribution will be symmetrical and can be represented by the Gaussian distribution (Fig. 5.4). The Gaussian model is widely applicable to many applications in counting statistics.

It should be emphasized that the distribution of all of the above models becomes identical for processes with a small individual success probability p and with a large enough number of trials such that the expected mean number of successes \bar{x} is large. This is demonstrated in Fig. 5.4 for $p = 0.0001$ and $\bar{x} = 100$.

5.3.2. Binomial distribution

Binomial distribution is the most general of the statistical models discussed. If n is the number of trials for which each trial has a success probability p , then the predicted probability of counting exactly x successes is given by:

$$P(x) = \frac{n!}{(n-x)!x!} p^x (1-p)^{n-x} \tag{5.12}$$

$P(x)$ is the predicted probability distribution function, and is defined only for integer values of n and x . The value of $n!$ (n factorial) is the product of integers up to n , that is $1 \times 2 \times 3 \times \dots \times n$. The values of $x!$ and $(n-x)!$ are similarly calculated.

The properties of the binomial distribution are as follows:

— The distribution is normalized:

$$\sum_{x=0}^n P(x) = 1 \tag{5.13}$$

— The mean value \bar{x} of the distribution using Eq. (5.8) is given by:

$$\bar{x} = \sum_{x=0}^n xP(x) \tag{5.14}$$

If Eq. (5.12) is substituted for $P(x)$, the mean value \bar{x} of the distribution is given by:

$$\bar{x} = pn \quad (5.15)$$

The sample variance for a set of experimental data has been defined by Eq. (5.9). By analogy, the predicted variance σ^2 is given by:

$$\sigma^2 = \sum_{x=0}^{\infty} (x - \bar{x})^2 P(x) \quad (5.16)$$

If Eq. (5.12) is substituted for $P(x)$, the predicted variance σ^2 of the distribution will be:

$$\sigma^2 = np(1 - p) \quad (5.17)$$

If Eq. (5.15) is substituted for np :

$$\sigma^2 = \bar{x}(1 - p) \quad (5.18)$$

The standard deviation σ is the square root of the predicted variance σ^2 :

$$\sigma = \sqrt{np(1 - p)} = \sqrt{\bar{x}(1 - p)} \quad (5.19)$$

The fractional standard deviation σ_F is given by:

$$\sigma_F = \frac{\sqrt{np(1 - p)}}{np} = \sqrt{\frac{(1 - p)}{np}}$$

$$\sigma_F = \frac{\sqrt{\bar{x}(1 - p)}}{\bar{x}} = \sqrt{\frac{(1 - p)}{\bar{x}}} \quad (5.20)$$

Equation (5.19) predicts the amount of fluctuation inherent in a given binomial distribution in terms of the basic parameters, namely the number of trials n and the success probability p , where $\bar{x} = pn$.

5.3.2.1. Application example of binomial distribution

The operation of a scintillation detector (Section 6.4) is considered. It consists of a scintillation crystal mounted on a PMT in a light tight construction. Firstly, when a γ ray interacts with the crystal, it generates n light photons.

Secondly, the light photons then eject x electrons from the photomultiplier photocathode. Thirdly, these electrons are then multiplied to form a pulse that can be further processed. For each γ ray that interacts with the scintillator, the number of light photons n , electrons ejected x and multiplication vary statistically during the detection of the different γ rays. This variation determines the energy resolution of the system.

In this example, the second stage is illustrated, that is the ejection of electrons from the photocathode. The variation or the standard deviation and fractional standard deviation for the number of electrons x that are ejected can be calculated using the binomial distribution as is given by Eqs (5.19) and (5.20).

The typical values for a scintillation counter are as follows. It is assumed that the 142 keV γ rays emitted by ^{99m}Tc are being counted. It is further assumed that it uses 100 eV to generate a light photon in the scintillation crystal when a γ ray interacts with the crystal. Therefore, if all of the energy of a single 142 keV photon is absorbed, $n = 142\,000/100 = 1420$ light photons will be emitted. It is assumed that these light photons fall on the photocathode of the PMT to generate x electrons for each γ ray absorbed. It is further assumed that five light photons are required to eject one electron.

For the binomial distribution, the probability of a light photon ejecting an electron is $p = 1/5$ and the number of trials n will be the number of light photons generated for each γ ray. This will be 1420. Equation (5.15) can be used to calculate the predicted mean number of electrons ejected for each γ ray:

$$\bar{x} = pn = \frac{1}{5} \times 1420 = 284 \text{ electrons}$$

The standard deviation (Eq. (5.19)) and relative standard deviation (Eq. (5.20)) can be calculated using the binomial distribution:

$$\sigma = \sqrt{\bar{x}(1-p)} = \sqrt{284(1-1/5)} = 15$$

and

$$\sigma_F = \sqrt{\frac{(1-p)}{\bar{x}}} = \sqrt{\frac{(1-1/5)}{284}} = 0.053 \quad (5.21)$$

Therefore, the contribution to the overall standard deviation at the electron ejection stage at the photocathode is 5.3%. The variation in the number of electrons will influence the pulse height obtained for each γ ray. The variation in the pulse height during the detection of γ rays will determine the width of the

photopeak (Fig. 5.1) and the energy resolution of the system (Sections 5.7.1 and 6.4).

5.3.3. Poisson distribution

Many binary processes can be characterized by a low probability of success for each individual trial. This includes nuclear counting and imaging applications in which large numbers of radionuclides make up the sample or number of trials, but a relatively small fraction of these give rise to recorded counts. Similarly, during imaging, many γ rays are emitted by the administered imaging radionuclide, for every one that interacts with the tissue. In addition, during nuclear counting, many γ rays strike the detector for every single recorded interaction.

Under these conditions, the approximation that the probability p is small ($p \ll 1$) will hold and some mathematical simplifications can be applied to the binomial distribution. The binomial distribution reduces to the form:

$$P(x) = \frac{(pn)^x e^{-pn}}{x!} \quad (5.22)$$

The relation $pn = \bar{x}$ holds for this distribution as well as for the binomial distribution:

$$P(x) = \frac{(\bar{x})^x e^{-\bar{x}}}{x!} \quad (5.23)$$

Equation (5.23) is the form of the Poisson distribution.

For the calculation of binomial distribution, two parameters are required: the number of trials n and the individual success probability p . It is noted from Eq. (5.23) that only one parameter, the mean value \bar{x} , is required. This is a very useful simplification because using only the mean value of the distribution, all other values of the Poisson distribution can be calculated. This is of great help for processes in which the mean value can be measured or estimated, but for which there is no information about either the individual probability or size of the sample. This is the case in nuclear counting and imaging.

The properties of the Poisson distribution are as follows. The Poisson distribution is a normalized frequency distribution (see Eqs (5.6) and (5.7)):

$$\sum_{x=0}^n P(x) = 1 \quad (5.24)$$

The mean value or first moment for the Poisson distribution is calculated by inserting the Poisson distribution (Eq. (5.22)) into the equation to calculate the mean for a frequency distribution (Eq. (5.8)):

$$\bar{x} = \sum_{x=0}^{\infty} xP(x) = pn \quad (5.25)$$

This is the same result as was obtained for the binomial distribution.

The predicted variance of the Poisson distribution differs from that of the binomial distribution and can be derived from Eqs (5.9) and (5.22):

$$\sigma^2 = \sum_{x=0}^{\infty} (x - \bar{x})^2 P(x) = pn \quad (5.26)$$

From the result of Eq. (5.26), the predicted variance is reduced to the important general equation:

$$\sigma^2 = \bar{x} \quad (5.27)$$

The predicted standard deviation is the square root of the predicted variance (Eq. (5.4)):

$$\sigma = \sqrt{\sigma^2} = \sqrt{\bar{x}} \quad (5.28)$$

The predicted standard deviation of any Poisson distribution is just the square root of the mean value that characterizes the same distribution.

The predicted fractional standard deviation σ_F (fractional error or coefficient of variation) is the standard deviation divided by the mean (Eq. (5.5)):

$$\sigma_F = \frac{\sigma}{\bar{x}} = \frac{1}{\sqrt{\bar{x}}} = \frac{1}{\sigma} \quad (5.29)$$

The fractional standard deviation is the inverse of the square root of the mean value of the distribution.

Equations (5.28) and (5.29) are important equations and frequently find application in nuclear detection and imaging.

5.3.4. Normal distribution

The Poisson distribution holds as a mathematical simplification to the binomial distribution within the limit $p < 1$. If, in addition, the mean value of the distribution is large (>30), additional simplification can generally be carried out which leads to a normal or Gaussian distribution:

$$P(x) = \frac{1}{\sqrt{2\pi\bar{x}}} e^{-\frac{(x-\bar{x})^2}{2\bar{x}}} \quad (5.30)$$

The distribution function is only defined for integer values of x .

Figure 5.4 for $\bar{x} = 100$ and $p = 0.0001$ demonstrates that for these values the normal distribution is identical to the Poisson and binomial distributions. The normal distribution is always symmetrical or ‘bell-shaped’ (Fig. 5.4). It shares the following properties with the Poisson distribution:

- It is normalized (see Section 5.2.1.2 and Eqs (5.6) and (5.7)):

$$\sum_{x=0}^n P(x) = 1 \quad (5.31)$$

- The distribution is characterized by a single parameter $\bar{x} = pn$.
- The predicted variance of the normal distribution is given by the mean of x :

$$\sigma^2 = \bar{x} \quad (5.32)$$

- The predicted standard deviation is the square root of the predicted variance (Eq. (5.4)):

$$\sigma = \sqrt{\sigma^2} = \sqrt{\bar{x}} \quad (5.33)$$

- The predicted fractional standard deviation σ_F (fractional error or coefficient of variation) is the standard deviation divided by the mean (Eq. (5.5)):

$$\sigma_F = \frac{\sigma}{\bar{x}} = \frac{1}{\sqrt{\bar{x}}} = \frac{1}{\sigma} \quad (5.34)$$

The fractional standard deviation is the inverse of the square root of the mean value of the distribution.

5.3.4.1. Continuous normal distribution: confidence intervals

In experiments where the sample size is small, there are only a few discrete outcomes. As the sample size increases, so does the number of possible sample outcomes. As the sample size approaches infinity, there is, in effect, a continuous distribution of outcomes. In addition, some random variables, such as height and weight, are essentially continuous and have continuous distributions. In these situations, the probability of a single event is not small as was assumed for the discrete Poisson and normal distributions, and the equation $\sigma = \sqrt{\bar{x}}$ does not apply. The continuous normal distribution is given by:

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\bar{x}}{\sigma}\right)^2} \quad (5.35)$$

The properties (Fig. 5.5) of the continuous normal distribution are:

- It is a continuous, symmetrical curve with both tails extending to infinity.
- All three measures of central tendency, mean, median and mode, are identical.
- It is described by two parameters: the arithmetic mean \bar{x} and the standard deviation σ .

The mean \bar{x} determines the location of the centre of the curve and the standard deviation σ represents the spread around the mean.

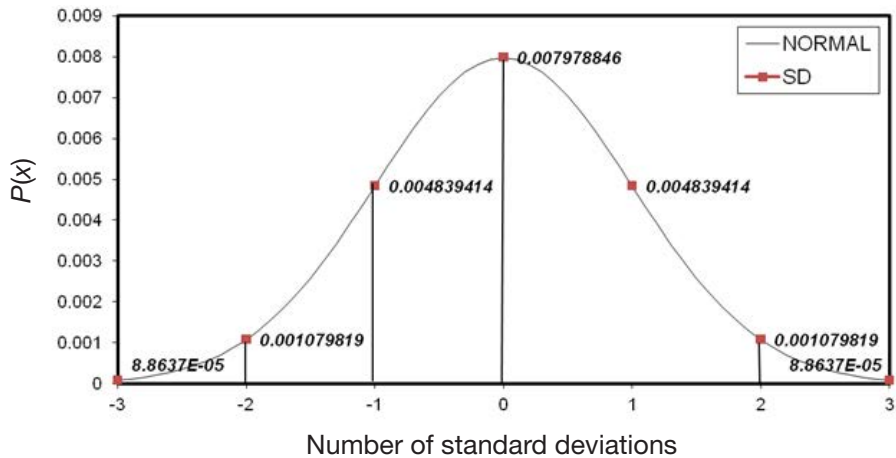


FIG. 5.5. The continuous normal distribution indicating the probability levels at different standard deviations (SDs) from the mean.

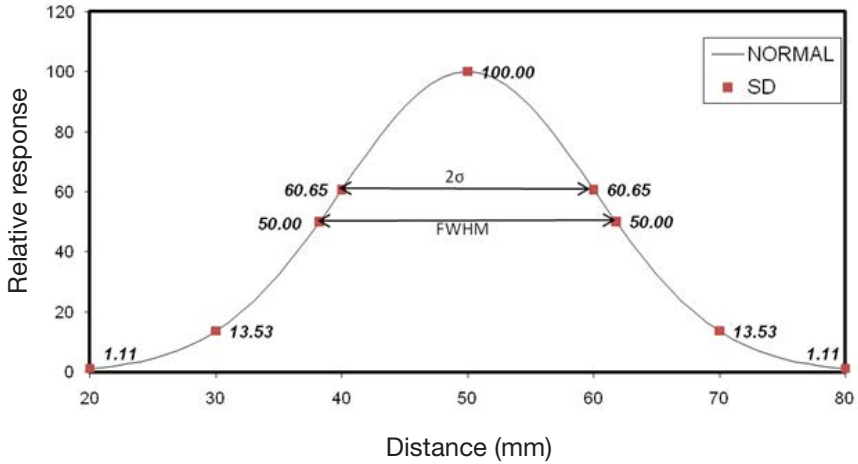


FIG. 5.6. Line source response curve obtained from a scintillation camera fitted to a normal distribution model. Image resolution is measured as the distance of the full width at half maximum (FWHM) of the percentage response. The standard deviation (SD) σ is the half width at a percentage response of 60.65%.

All continuous normal distributions have the property that between the mean and one standard deviation 68% is included on either side, between the mean and two standard deviations 95%, and between the mean and three standard deviations 99.7% of the total area under the curve.

5.3.4.2. Continuous normal distribution: applications in medical physics

The normal distribution is often used in radionuclide measurements and imaging to fit to experimental data. In this case, the equation is modified as follows:

$$P(x) = 100e^{-\frac{1}{2}\left(\frac{x-\bar{x}}{\sigma}\right)^2} \tag{5.36}$$

where the maximum value of the distribution at \bar{x} is normalized to 100.

The spatial resolution of imaging devices such as scintillation cameras and positron emission tomography equipment is determined as the full width at half maximum (FWHM) response of a normal distribution fitted to a point or line spread function (Fig. 5.6). The FWHM of the imaging device used in Fig. 5.6

was 23.6 mm. The relation for a normal distribution between the FWHM and standard deviation σ can be derived by setting $P(x) = 50$ and solving Eq. (5.36):

$$\text{FWHM} = 2.355\sigma \quad (5.37)$$

For the imaging system used in Fig. 5.6, the standard deviation $\sigma = 10$ mm. The value of the response $P(x)$ is 60.65% at a distance of $\sigma = \bar{x}$ (Eq. (5.36)). The value of the standard deviation σ can, therefore, also be obtained from the measured percentage response curve by finding the x value at a percentage response of 60.65% (Fig. 5.6).

In radionuclide energy spectroscopy, the photopeak distribution can be fitted to a normal distribution (Fig. 5.1). The energy resolution of scintillation detectors is expressed as the FWHM of the photopeak distribution divided by the photopeak energy E . The energy spectrum in medical physics applications is measured in kiloelectronvolts or megaelectronvolts. The fractional energy resolution R_E is:

$$R_E = \frac{\text{FWHM}}{E} = \frac{2.355\sigma}{E} \quad (5.38)$$

5.4. ESTIMATION OF THE PRECISION OF A SINGLE MEASUREMENT IN SAMPLE COUNTING AND IMAGING

5.4.1. Assumption

A valuable application of counting statistics applies to the case in which only a single measurement of a particular quantity is available and the uncertainty associated with that measurement is required. The square root of the sample variance σ should be a measure of the deviation of any one measurement from the true mean value and will serve as an index of the degree of precision that should be associated with a measurement from that set.

As only a single measurement is available, the sample variance cannot be calculated directly using Eqs (5.3) or (5.9) and must be estimated by analogy with an appropriate statistical model. The appropriate theoretical distribution can be matched to the available data if the measurement has been drawn from a population whose theoretical distribution function is predicted by either a Poisson or Gaussian distribution. As the value of the single measurement x is the

only information available, it is assumed that the mean of the distribution is equal to the single measurement:

$$\bar{x} \approx x \quad (5.39)$$

Having obtained an assumed value for \bar{x} , the entire predicted probability distribution function $P(x)$ is defined for all values of x .

The expected sample variance s^2 can be expressed in terms of the variance of the selected statistical model:

$$s^2 = \sigma^2 = \bar{x} \approx x \quad (5.40)$$

Therefore, the best estimate of the deviation σ from the true mean, which should typify a single measurement x , is given by:

$$\sqrt{s} = \sigma \approx \sqrt{x} \quad (5.41)$$

To illustrate the application of Eq. (5.41), it is assumed that the probability distribution function is Gaussian with a large value for the measurement x . The range of values $x \pm \sigma$ or $x \pm \sqrt{x}$ will contain the true mean with 68% probability.

If it is assumed that there is a single measurement $x = 100$, then:

$$\sigma \approx \sqrt{x} = \sqrt{100} = 10$$

In Table 5.3, the various options available in quoting the uncertainty to be associated with the single measurement are shown. The conventional choice is to quote the measurement x plus or minus the standard deviation σ or 100 ± 10 . This interval is expected to contain the true mean \bar{x} with a probability of 68%. The probability that the true mean is included in the range can be increased by expanding the interval associated with the measurement as is shown in Table 5.3. For example, to achieve a 99% probability that the true mean is included, the interval must be expanded by 2.58σ . In the example, the range is then 100 ± 25.8 .

When errors are reported, the associated probability level should be stated in the report under methods.

TABLE 5.3. EXAMPLES OF ERROR INTERVALS FOR A SINGLE MEASUREMENT $x = 100$

Interval (relative σ)	Interval (values)	Probability that the true mean \bar{x} is included (%)
$x \pm 0.67\sigma$	93.3–106.7	50
$x \pm 1.00\sigma$	90.0–110.0	68
$x \pm 1.64\sigma$	83.6–116.4	90
$x \pm 2.00\sigma$	80.0–120.0	95
$x \pm 2.58\sigma$	74.2–125.8	99
$x \pm 3.00\sigma$	70.0–130.0	99.7

5.4.2. The importance of the fractional σ_F as an indicator of the precision of a single measurement in sample counting and imaging

The relation between the precision and a single counting measurement x is given by Eq. (5.40). The precision, expressed as the standard deviation σ , will increase proportionally to the square root of the measurement x . Thus, if the value of the single measurement x increases, the standard deviation will also increase. The increase in the standard deviation will be smaller than that of the measurement x . The relation between the standard deviation and the single measurement is best demonstrated by calculating the relative or fractional standard deviation σ_F :

$$\sigma_F = \frac{\sigma}{x} = \frac{\sqrt{x}}{x} = \frac{1}{\sqrt{x}} \quad (5.42)$$

Thus, the recorded number of counts or the value of the single measurement x completely determines the relative precision. The relative precision decreases as the number of counts increases. Therefore, to achieve a required relative precision, a minimum number of counts must be accumulated.

The following example illustrates the important relation between the relative precision and the number of counts recorded. If 100 counts are recorded, the relative standard deviation is 10%. If 10 000 counts are recorded, the relative standard deviation reduces to 1%. This example demonstrates the importance of acquiring enough counts to meet the required precision.

It is easier to achieve the required precision when samples in counting tubes are measured than when *in vivo* measurements on patients are performed. The

single measurement from a high count rate radioactive sample in a counting tube will be obtained in a short time. However, if a low activity sample is measured, the measurement time will have to be increased to achieve the desired precision. The desired precision can be conveniently obtained by using automatic sample counters. These counters can be set to stop counting after a preset time or preset counts have been reached. By choosing the preset count option, the desired precision can be achieved for each sample.

The acquisition time of in vivo measurements using collimated detector probes, such as thyroid iodine uptake studies or imaging studies, can often not be increased to achieve the desired precision as a result of patient movement. In these single measurements, high sensitivity radiation detectors or a higher, but acceptable radioactive dose, can be selected.

The precision of a single measurement is very important during radionuclide imaging. If the number of counts acquired in a picture element or pixel is low, a low precision is obtained. There will then be a wide range of fluctuations between adjacent pixels. As a result of the poor quality of the images, it would only be possible to identify large defect volumes or defects with a high contrast. To detect a defect, the measured counts from the defect must lie outside the range of the background measurement plus or minus two standard deviations ($x \pm 2\sigma$). During imaging, the number of counts measured in a target volume will be determined by the acquisition time, activity within the target volume and the sensitivity of the measuring equipment. The sensitivity of imaging equipment can be increased by increasing the FWHM spatial resolution. There is a trade-off between single sample counting precision and the spatial resolution of the imaging device to obtain images that would provide the maximum diagnostic value during visual interpretation of the images by nuclear medicine physicians.

Counting statistics are also very important during image quantification such as measuring renal function, left ventricular ejection fraction and tumour uptake. During quantification, the accumulated counts by an organ or within a target volume have to be accurately determined. In quantification studies, the background activity, attenuation and scatter contributions have to be corrected. These procedures further reduce the precision of quantification.

5.4.3. Caution on the use of the estimate of the precision of a single measurement in sample counting and imaging

All conclusions are based on the measurement of a counted number of success (number of heads in coin tossing). In nuclear measurements or imaging, the estimate of the precision of a single measurement by using $\sigma = \sqrt{x}$ can only be applied if x represents a counted number of success, that is the number of events recorded in a given observation time.

The estimate of the precision of a single measurement by using $\sigma = \sqrt{x}$ cannot be used if x is not a directly measured count. For example, the association does not apply to:

- Counting rates;
- Sums or differences of counts;
- Averages of independent counts;
- Pixel counts following tomographic image reconstruction;
- Any derived quantity.

In these cases, the quantity is calculated as a function of the number of counts recorded. The error to be associated with that quantity must be calculated according to the error propagation methods outlined in the next section.

5.5. PROPAGATION OF ERROR

The preceding section described methods for estimating random error or the precision of a single measurement during nuclear measurements or imaging. Most procedures in nuclear medicine involve multiple nuclear measurements and imaging procedures for the calculation of results such as thyroid iodine uptake, ejection fraction, renal clearance, blood volume or red cell survival time, on which clinical diagnosis is based. Similarly, internal dosimetry is performed using nuclear measurements and imaging data. To estimate the corresponding precision in the derived quantity, how the error associated with the initial measurements propagates through the calculations that were performed to arrive at the required result has to be followed. This is done by applying the error of propagation formulas. The variables used in the calculation of errors must be independent to avoid effects of correlation. It is assumed that the error in nuclear measurements arises only from random fluctuations in the decay rate and is statistically independent of other errors.

The error of propagation formulas applies to measurements that are obtained from a continuous distribution as well as to Poisson and discrete normal distributions. The measurements from continuous distributions will be represented by x_1, x_2, x_3, \dots with variances of $\sigma(x_1)^2, \sigma(x_2)^2, \sigma(x_3)^2, \dots$. These equations can be used to estimate precision in measurements such as height and weight.

Discrete nuclear measurements with Poisson or normal distribution are represented by N_1, N_2, N_3, \dots with variances of $\sigma(N_1)^2, \sigma(N_2)^2, \sigma(N_3)^2, \dots$ or N_1, N_2, N_3, \dots .

5.5.1. Sums and differences

The product x_s of the sums or difference of a series of measurements with a continuous normal distribution is given by:

$$x_s = x_1 \pm x_2 \pm x_3 \dots \quad (5.43)$$

The variance of x_s is given by:

$$\sigma(x_1 \pm x_2 \pm x_3 \dots)^2 = \sigma(x_1)^2 + \sigma(x_2)^2 + \sigma(x_3)^2 \dots \quad (5.44)$$

The standard deviation is given by:

$$\sigma(x_1 \pm x_2 \pm x_3 \dots) = \sqrt{\sigma(x_1)^2 + \sigma(x_2)^2 + \sigma(x_3)^2 \dots} \quad (5.45)$$

The fractional standard deviation is given by:

$$\sigma_F(x_1 \pm x_2 \pm x_3 \dots) = \frac{\sqrt{\sigma(x_1)^2 + \sigma(x_2)^2 + \sigma(x_3)^2 \dots}}{x_1 \pm x_2 \pm x_3 \dots} \quad (5.46)$$

For counting measurements or measurements with a Poisson or discrete normal distribution, the variance is given by:

$$\sigma(N_1 \pm N_2 \pm N_3 \dots)^2 = N_1 \pm N_2 \pm N_3 \dots \quad (5.47)$$

The standard deviation is given by:

$$\sigma(N_1 \pm N_2 \pm N_3 \dots) = \sqrt{N_1 + N_2 + N_3 \dots} \quad (5.48)$$

The fractional standard deviation is given by:

$$\sigma_F(N_1 \pm N_2 \pm N_3 \dots) = \frac{\sqrt{N_1 + N_2 + N_3 \dots}}{N_1 \pm N_2 \pm N_3 \dots} \quad (5.49)$$

These equations apply to mixed combinations of sums and differences.

TABLE 5.4. UNCERTAINTY AFTER SUMMING AND SUBTRACTING COUNTS

	$N_2 \ll N_1$			$N_2 \approx N_1$		
	N	σ	σ_F	N	σ	σ_F
N_1	500	22.4	0.0447	500	22.4	0.0447
N_2	10	3.2	0.3162	450	21.2	0.0471
$N_1 - N_2$	490	22.6	0.0461	50	30.8	0.6164
$N_1 + N_2$	510	22.6	0.0443	950	30.8	0.0324

The influence on the standard deviation and fractional standard deviation of summing and subtracting values N_1 and N_2 is demonstrated in Table 5.4. The following conclusions can be drawn:

- The standard deviation σ for $N_1 - N_2$ and $N_1 + N_2$ is the same for the same values of N_1 and N_2 , but the fractional standard deviation σ_F is different;
- The fractional standard deviation for differences is large when the differences between the values are small.

This is the reason why it is important to limit the background to a value as low as possible in counting procedures. In imaging, when scatter or background correction is performed by subtraction, image quality deteriorates as a result of the increased uncertainty in the pixel values.

5.5.2. Multiplication and division by a constant

We define:

$$x_M = Ax \tag{5.50}$$

where A is a constant.

Then:

$$\sigma_M = A\sigma_x \tag{5.51}$$

and

$$\sigma_F = \frac{A\sigma_x}{Ax} = \frac{\sigma_x}{x} \quad (5.52)$$

For counting measurements or measurements with a Poisson or discreet normal distribution, the following applies:

$$x_M = AN \quad (5.53)$$

Then:

$$\sigma_M = A\sqrt{N} \quad (5.54)$$

and

$$\sigma_F = \frac{1}{\sqrt{N}} \quad (5.55)$$

Similarly, if:

$$x_D = \frac{x}{B} \quad (5.56)$$

where B is also a constant:

$$\sigma_M = \frac{\sigma_x}{B} \quad (5.57)$$

and

$$\sigma_F = \frac{\sigma_x B}{B x} = \frac{\sigma_x}{x} \quad (5.58)$$

For counting measurements or measurements with a Poisson or discreet normal distribution, the following apply:

$$x_D = \frac{N}{B} \quad (5.59)$$

$$\sigma_M = \frac{\sqrt{N}}{B} \quad (5.60)$$

and

$$\sigma_F = \frac{1}{\sqrt{N}} \tag{5.61}$$

It should be noted that multiplying (Eqs (5.52) and (5.55)) or dividing (Eqs (5.58) and (5.61)) a value by a constant does not change the fractional standard deviation.

5.5.3. Products and ratios

The uncertainty in the product or ratio of a series of measurements $x_1, x_2, x_3...$ is expressed in terms of the fractional uncertainties in the individual results, $\sigma_F(x_1), \sigma_F(x_2), \sigma_F(x_3)...$

The product x_p of the products or ratios of a series of measurements with a continuous normal distribution is given by:

$$x_p = x_1 \times x_2 \times x_3 \times \dots \dots \dots \tag{5.62}$$

The notation \times means $x_1 \times x_2$ or $x_1 \div x_2$. These equations apply to mixed combinations of sums and differences.

The fractional variance of x_p is given by:

$$\sigma_F(x_1 \times x_2 \times x_3 \times \dots \dots)^2 = \sigma_F(x_1)^2 + \sigma_F(x_2)^2 + \sigma_F(x_3)^2 \dots \dots \tag{5.63}$$

The fractional standard deviation is given by:

$$\sigma_F(x_1 \times x_2 \times x_3 \times \dots \dots) = \sqrt{\sigma_F(x_1)^2 + \sigma_F(x_2)^2 + \sigma_F(x_3)^2 \dots \dots} \tag{5.64}$$

The standard deviation is given by:

$$\sigma(x_1 \times x_2 \times x_3 \times \dots \dots) = \sqrt{\sigma_F(x_1)^2 + \sigma_F(x_2)^2 + \sigma_F(x_3)^2 \dots} \times (x_1 \times x_2 \times x_3 \times \dots \dots) \tag{5.65}$$

For counting measurements or measurements with a Poisson or discreet normal distribution, the product or ratio is given by:

$$N_p = N_1 \times N_2 \times N_3 \times \dots \dots \tag{5.66}$$

The fractional variance of N_p is given by:

$$\sigma_F(N_1 \times N_2 \times N_3 \times \dots \dots)^2 = \frac{1}{N_1} + \frac{1}{N_2} + \frac{1}{N_3} + \dots \dots \tag{5.67}$$

The fractional standard deviation is given by:

$$\sigma_F(N_1 \times N_2 \times N_3 \times \dots) = \sqrt{\frac{1}{N_1} + \frac{1}{N_2} + \frac{1}{N_3} + \dots} \quad (5.68)$$

The standard deviation is given by:

$$\sigma(N_1 \times N_2 \times N_3 \times \dots) = \sqrt{\frac{1}{N_1} + \frac{1}{N_2} + \frac{1}{N_3} + \dots} (N_1 \times N_2 \times N_3 \times \dots) \quad (5.69)$$

5.6. APPLICATIONS OF STATISTICAL ANALYSIS

5.6.1. Multiple independent counts

5.6.1.1. Sum of multiple independent counts

If it is supposed that there are n repeated counts from the same source for equal counting times and the results of the measurements are $N_1, N_2, N_3, \dots, N_n$ and their sum is N_s , then:

$$N_s = N_1 + N_2 + N_3 \dots \quad (5.70)$$

According to the propagation of error for sums and Eq. (5.48):

$$\sigma_{N_s} = \sqrt{N_1 + N_2 + N_3 \dots} = \sqrt{N_s} \quad (5.71)$$

The results show that the standard deviation for the sum of all counts is the same as if the measurement had been carried out by performing a single count, extending over the period represented by all of the counts.

5.6.1.2. Mean value of multiple independent counts

If the mean value \bar{N} of the n independent counts referred to in the previous section is calculated, then:

$$\bar{N} = \frac{N_s}{n} \quad (5.72)$$

Equation (5.72) is an example of dividing an error-associated quantity \bar{N} by a constant n . Equation (5.51), therefore, applies and the standard deviation of the mean or standard error is given by:

$$\sigma_{\bar{N}} = \frac{\sigma_{N_s}}{n} = \frac{\sqrt{N_s}}{n} = \frac{\sqrt{n\bar{N}}}{n} = \sqrt{\frac{\bar{N}}{n}} \quad (5.73)$$

It should be noted that the standard deviation for a single measurement N_i (Eq. (5.41)) is $\sigma_{N_i} = \sqrt{N_i}$.

A typical count will not differ greatly from the mean $N_i \approx \bar{N}$. Thus, the mean value based on n independent counts will have an expected error that is smaller by a factor of \sqrt{n} compared with any single measurement on which the mean is based. To improve the statistical precision of a given measurement by a factor of two, the counting time must, therefore, be increased four times.

5.6.2. Standard deviation and relative standard deviation for counting rates

If N counts are accumulated over time t , then the counting rate R is given by:

$$R = \frac{N}{t} \quad (5.74)$$

In the above equation, it is assumed that the time t is measured with a very small uncertainty, so that t can be considered a constant. The calculation of the uncertainty associated with the counting rate is an application of the propagation of errors, multiplying by a constant (Eq. (5.60)):

$$\sigma_R = \frac{\sigma_x}{t} = \frac{\sqrt{N}}{t} = \sqrt{\frac{R}{t}} \quad (5.75)$$

The fractional standard deviation is calculated using Eq. (5.61):

$$\sigma_F = \frac{\sigma_x}{iR} = \frac{\sqrt{N}}{tR} = \sqrt{\frac{1}{tR}} \quad (5.76)$$

The above equations illustrate the calculation of uncertainties if calculations are required to obtain a value, and the equation for a single value (Section 5.3) cannot be applied. The following example illustrates the use of the equations.

5.6.2.1. Example: comparison of error of count rates and counts accumulated

The activity of two samples is measured. Sample 1 is counted with a counter that is set to stop when a count of 10 000 is reached. It takes 100 s to reach 10 000 counts. Sample 2 is counted using an automatic sample changer. The activity of the sample is given as 10 000 counts per second (cps) and the sample was counted for 100 s.

Calculating the counting error associated with the measurements of samples 1 and 2:

Sample 1:

The counts acquired: $N = 10\,000$ counts

Standard deviation (Eq. (5.41)): $\sigma_N = 100$ counts

Fractional standard deviation (Eq. (5.42)): $\sigma_F = 0.01 = 1\%$

Sample 2:

The count rate: 10 000 cps

Standard deviation (Eq. (5.75)): $\sigma_R = \sqrt{\frac{10\,000}{100}} = 10$ cps

Fractional standard deviation (Eq. (5.76)):

$$\sigma_F = \sqrt{\frac{1}{10\,000 \times 100}} = 0.001 = 0.1\%$$

Although the counts acquired for sample 1 and the count rate of sample 2 were numerically the same, the uncertainties associated with the measurements were very different. When calculations on counts are performed, it must be determined whether the value is a single value or whether it is a value that has been obtained by calculation.

5.6.3. Effects of background counts

Background counts are those counts that do not originate from the sample or target volume or are unwanted counts such as scatter. The background counts during sample counting consist of electronic noise, detection of cosmic rays, natural radioactivity in the detector, and down scatter radioactivity from non-target radionuclides in the sample. During in vivo measurements, such as measurement of thyroid iodine uptake or left ventricular ejection fraction, radiation from non-target tissue will also contribute to background. Scattered

radiation from target as well as non-target tissue will influence quantification and will be included in the background. To obtain the true net counts, the background is subtracted from the gross counts accumulated. The uncertainty of the true target counts can be calculated using Eqs (5.48) and (5.49), and the uncertainty of true count rates can be calculated using Eqs (5.75) and (5.76).

If the background count is N_b , and the gross counts of the sample and background is N_g , then the net sample count N_s is:

$$N_s = N_g - N_b \quad (5.77)$$

The standard deviation for N_s counts is given by Eq. (5.48):

$$\sigma(N_s) = \sqrt{N_g + N_b} \quad (5.78)$$

The fractional standard deviation for N_s counts is given by Eq. (5.49):

$$\sigma_F(N_s) = \frac{\sqrt{N_g + N_b}}{N_g - N_b} \quad (5.79)$$

If the background count rate is R_b , acquired in time t_b , and the gross count rate of the sample and background is R_g , acquired in time t_g , then the net sample count rate R_s is:

$$R_s = R_g - R_b \quad (5.80)$$

The standard deviation for a count rate R_s is given by Eqs (5.45) and (5.75):

$$\sigma(R_s) = \sqrt{\frac{R_g}{t_g} + \frac{R_b}{t_b}} \quad (5.81)$$

The fractional standard deviation for a count rate R_s is given by Eqs (5.46) and (5.76):

$$\sigma_F(R_s) = \frac{\sqrt{\frac{R_g}{t_g} + \frac{R_b}{t_b}}}{R_g - R_b} \quad (5.82)$$

If the same counting time t is used for both sample and background measurement:

$$\sigma(R_s) = \frac{\sqrt{R_g + R_b}}{\sqrt{t}} \quad (5.83)$$

and

$$\sigma_F(R_s) = \frac{\sqrt{R_g + R_b}}{\sqrt{t}(R_g - R_b)} \quad (5.84)$$

5.6.3.1. Example: error in net target counts following background correction

The following example illustrates the application to determine the uncertainty in the measurement of target volume counts following background correction. A planar image of the liver is acquired for the detection of tumours. Two equal sized ROIs, ROI1 and ROI2, were selected to cover the areas of the two potential tumours. The gross counts N_g in ROI1 were 484 counts (Table 5.5) and in ROI2 484 counts. The background counts N_b selected over normal tissue of the same area as for the gross counts were 441 and 169 counts. How to calculate the uncertainties in the tumor volume net counts is presented.

The difference and error associated with the difference (Eq. (5.77) – Eq. (5.79)) when $N_g \approx N_b$ are:

$$N_g - N_b = 484 - 441 = 43 \text{ counts}$$

$$\sigma(N_g - N_b) = \sqrt{484 + 441} = 30.4 \text{ counts}$$

$$\sigma_F(N_g - N_b) = \frac{\sqrt{484 + 441}}{484 - 441} = 0.7073$$

$$\sigma_P(N_g - N_b) = 70.7\%$$

The influence on the standard deviation and fractional standard deviation of background correction for $N_g \approx N_b$ and $N_g \gg N_b$ is demonstrated in Table 5.5. The following conclusion can be drawn: the fractional σ_F and percentage σ_P standard deviations significantly increase when the background increases relative to the net counts.

This is the reason why it is important in measurements of radioactivity to acquire as many counts as possible to decrease the uncertainty in detection of target volume radioactivity. The following example illustrates the application to determine the uncertainty in the measurement of target volume count rate following background correction.

TABLE 5.5. CALCULATION OF UNCERTAINTIES IN COUNTS AS A RESULT OF BACKGROUND CORRECTION

$N_g \approx N_b$					$N_g \gg N_b$				
Source	Counts	σ counts	σ_F	σ_P (%)	Source	Counts	σ counts	σ_F	σ_P (%)
N_g	484	22.0	0.0455	4.5	N_g	484	22.0	0.0455	4.5
N_b	441	21.0	0.0476	4.8	N_b	169	13.0	0.0769	7.7
N_s	43	30.4	0.7073	70.7	N_s	315	25.6	0.0811	8.1
$3\sigma(N_s)$	91	Counts	Not significant		$3\sigma(N_s)$	77	Counts	Significant	

5.6.3.2. Example: error in net target count rate following background correction

A planar image of the liver is acquired for the detection of tumours. Two equal sized ROIs, ROI1 and ROI2, were selected to cover the areas of the two potential tumours. The gross count rate R_g in ROI1 was 484 counts per minute (cpm) (Table 5.6) and in ROI2 484 cpm. The background count rates R_b selected over normal tissue of the same area as for the gross counts were 441 and 169 cpm. The acquisition time of the image was 2 min. How to calculate the uncertainties in the tumor volume net counts is presented.

The difference and error associated with the difference (Eq. (5.80) – Eq. (5.82)) when $R_g \approx R_b$ are:

$$R_g - R_b = 484 - 441 = 43 \text{ cpm}$$

$$\sigma(R_g - R_b) = \sqrt{\frac{484}{2} + \frac{441}{2}} = 21.5 \text{ cpm}$$

$$\sigma_F(R_g - R_b) = \frac{\sqrt{\frac{484}{2} + \frac{441}{2}}}{484 - 441} = 0.5001$$

$$\sigma_P(R_g - R_b) = 50.0\%$$

The influence on the standard deviation and fractional standard deviation of background correction for $R_g \approx R_b$ and $R_g \gg R_b$ is demonstrated in Table 5.6.

Again, it is shown that the fractional standard deviation σ_F significantly increases when the background count rate increases relative to the net target count rate.

TABLE 5.6. CALCULATION OF UNCERTAINTIES IN COUNT RATES AS A RESULT OF BACKGROUND CORRECTION

$R_g \approx R_b$					$R_g \gg R_b$				
Source	Count rate (cpm)	σ (cpm)	σ_F	σ_P (%)	Source	Count rate (cpm)	σ_F (cpm)	σ_F	σ_P (%)
R_g	484	15.6	0.0321	3.2	R_g	484	15.6	0.0321	3.2
R_b	441	14.8	0.0337	3.4	R_b	169	9.2	0.0544	5.4
R_s	43	21.5	0.5001	50.0	R_s	315	18.1	0.0574	5.7
t	2	Minutes			t	2	Minutes		
$3\sigma(R_s)$	65	Not significant			$3\sigma(R_s)$	54	Significant		

5.6.4. Significance of differences between counting measurements

If N_1 and N_2 counts are measured in two counting measurements, the difference ($N_1 - N_2$) between the measured counts may be a result of random variations in the counting rate or may be as a result of an actual difference. The statistical significance of the difference is evaluated by comparing it to the expected random error expressed as the standard deviation σ_d of the difference. If $(N_1 - N_2) > 2\sigma(N_1 - N_2)$, there is a 5% chance that the difference is caused by random error (see Table 5.3). If:

$$N_1 - N_2 > 3\sigma(N_1 - N_2) \tag{5.85}$$

there is a 0.3% chance that the difference is caused by random error and this difference is considered significant.

The examples in the previous section to determine whether tumours were present following a liver scan illustrate the application to determine the significance of the difference between two counts (Table 5.5). The net counts and uncertainty over two tumour areas were calculated. Do the counts over the tumour areas significantly differ from the normal background area?

For the difference for $N_g \approx N_b$ (Table 5.5) to be significant, Eq. (5.85) must apply.

The difference of 43 cpm was less than the norm of $3\sigma(N_1 - N_2)$ and the difference is, therefore, not significant. It can be concluded with a smaller than 0.3% chance that there is not a tumour present.

An example when $N_g \gg N_b$ is also given in Table 5.5. In this case, the 315 cpm counts difference was larger than $3\sigma(N_1 - N_2)$ of 77 cpm. The difference in this case is significant. It can be concluded with a smaller than 0.3% chance that there is a tumour present.

The significance of differences between the counting rates of samples can also be calculated. Two counting rates, R_1 and R_2 , are acquired using counting times t_1 and t_2 .

The uncertainty associated with the difference is given by applying Eqs (5.45) and (5.75):

$$\sigma(R_1 - R_2) = \sqrt{\frac{R_1}{t_1} + \frac{R_2}{t_2}} \quad (5.86)$$

For the difference $R_1 - R_2$ to be significant:

$$R_1 - R_2 > 3\sigma(R_1 - R_2) \quad (5.87)$$

The examples in the previous section (Table 5.6) to determine whether tumours were present following a liver scan illustrate an application to determine the significance of the difference between two count rates. The net count rate and uncertainty over two tumour areas were calculated. Do the count rates over the tumour areas significantly differ from the normal background area?

For the difference for $R_g \approx R_b$ (Table 5.6) to be significant, Eq. (5.87) must apply.

The difference count rate of 43 cpm was less than the 65 cpm which is the norm of $3\sigma(R_1 - R_2)$ and the difference is, therefore, not significant. It can be concluded with a smaller than 0.3% chance that there is not a tumour present.

An example when $R_g \gg R_b$ is also given in Table 5.6. In this case, the difference of 315 cpm was larger than $3\sigma(R_1 - R_2)$ which was 54 cpm. The difference in this case is significant. It can be concluded with a smaller than 0.3% chance that there is a tumour present.

5.6.5. Minimum detectable counts, count rate and activity

According to Eq. (5.85), if the difference of two measurements is larger than three standard deviations, the difference is considered significant. Therefore,

the minimum net counts N_m that can be detected with 0.3% confidence is given by:

$$N_m = N_1 - N_2 = 3\sigma(N_1 - N_2) \quad (5.88)$$

or

$$N_m = N_g - N_b = 3\sigma(N_g - N_b) \quad (5.89)$$

Solving this equation for N_g will give the minimum detectable gross counts N_m :

$$N_g = \frac{(2N_b + 9) + \sqrt{72N_b + 81}}{2} \quad (5.90)$$

An approximation can be used by assuming that $N_g \approx N_b$ and:

$$N_g \approx N_b + 3\sqrt{2N_b} \quad (5.91)$$

The minimum detectable activity A_m can be calculated:

$$A_m = \frac{N_m}{tS} \quad (5.92)$$

where

S is the sensitivity of the detection system usually expressed as count rate per becquerel;

and t is the time that the background was counted.

5.6.5.1. Example: calculation of minimum activity that can be detected

A detector is to be used to detect ^{131}I in the thyroid of radiation workers. The background count was 441 counts measured over a period of 5 min. The acquisition time for the thyroid was also 5 min. The sensitivity of the counter was $0.1 \text{ counts} \cdot \text{s}^{-1} \cdot \text{Bq}^{-1}$. What is the minimum activity that can be detected?

From Eq. (5.90):

$$N_g = \frac{(2N_b + 9) + \sqrt{72N_b + 81}}{2} = \frac{(2 \times 441 + 9) + \sqrt{72 \times 441 + 81}}{2} = 535 \text{ counts}$$

It should be noted that $N_g - N_b = 94$ counts and $3\sigma(N_g - N_b) = 94$ as was specified in Eq. (5.85). The minimum detectable radioactivity is:

$$A_m = \frac{(535 - 441)}{5 \times 60 \times 0.1} = 3.124 \text{ Bq}$$

The minimum detectable net count rate R_m is given by Eq. (5.89):

$$R_m = R_g - R_b > 3\sigma(R_g - R_b) \quad (5.93)$$

Solving this equation for R_g gives the minimum detectable gross count rate R_m :

$$R_g = \frac{\left(2R_b + \frac{9}{t_g}\right) + \sqrt{\frac{36R_b}{t_g} + \frac{81}{t_g^2} + \frac{36R_b}{t_b}}}{2} \quad (5.94)$$

An approximation can be used by assuming that $R_g \approx R_b$ and from Eqs (5.86) and (5.87):

$$R_g \approx R_b + 3\sqrt{\frac{R_b}{t_g} + \frac{R_b}{t_b}} \quad (5.95)$$

The minimum detectable activity A_m can be calculated:

$$A_m = \frac{R_m}{S} \quad (5.96)$$

where S is the sensitivity of the detection system usually expressed as count rate per becquerel.

5.6.5.2. Example 2: calculation of minimum activity that can be detected

A detector is to be used to detect ^{131}I in the thyroid of radiation workers. The background count rate was 441 cpm measured over a period of 5 min and the thyroid count rate was measured over 1 min. The sensitivity of the counter was $0.1 \text{ counts} \cdot \text{s}^{-1} \cdot \text{Bq}^{-1}$. What is the minimum activity that can be detected?

From Eq. 5.94:

$$R_g = \frac{\left(2 \times 441 + \frac{9}{1}\right) + \sqrt{36 \times \frac{441}{1} + \frac{81}{1^2} + 36 \times \frac{441}{5}}}{2} = 515 \text{ cpm}$$

It should be noted that $R_g - R_b = 74$ cpm and $3\sigma(R_g - R_b) = 74$ cpm as was specified in Eq. (5.93). The minimum detectable radioactivity is:

$$A_m = \frac{(515 - 441)}{0.1 \times 60} = 12.28 \text{ Bq}$$

5.6.6. Comparing counting systems

It was concluded in Section 5.3.1 that a large number of counts have smaller uncertainties expressed as the fractional standard deviation. In Section 5.6.3, it was shown that if background counts increase, the uncertainty of the net counts expressed as fractional standard deviation rapidly increases. Thus, it is desirable to use a counting system with a high sensitivity and low background. However, when the detector sensitivity is increased, the system will also be more sensitive to background. The trade-off between sensitivity and background can be analysed as follows.

It is considered that results from systems 1 and 2 are compared. The acquisition times for gross and background counts are acquired over the same time. From Eq. (5.79):

$$\sigma_{F1}(N_{S1}) = \frac{\sqrt{N_{g1} + N_{b1}}}{N_{g1} - N_{b1}}$$

and

$$\sigma_{F2}(N_{S2}) = \frac{\sqrt{N_{g2} + N_{b2}}}{N_{g2} - N_{b2}}$$

The fractional uncertainties for the net sample counts obtained with the two systems are, therefore:

$$\frac{\sigma_{F1}(N_{S1})}{\sigma_{F2}(N_{S2})} = \frac{\frac{\sqrt{N_{g1} + N_{b1}}}{N_{g1} - N_{b1}}}{\frac{\sqrt{N_{g2} + N_{b2}}}{N_{g2} - N_{b2}}} \quad (5.97)$$

If $\frac{\sigma_{F1}(N_{S1})}{\sigma_{F2}(N_{S2})} < 1$, then system 1 is statistically the preferred system. If

$\frac{\sigma_{F1}(N_{S1})}{\sigma_{F2}(N_{S2})} > 1$, then system 2 is preferred.

Systems can be compared using the count rate and fractional standard deviation for the count rate R_s (Eq. (5.82)). To compare systems 1 and 2, the ratio of the fractional standard deviation is calculated:

$$\frac{\sigma_{F1}(R_{S1})}{\sigma_{F2}(R_{S2})} = \frac{\frac{\sqrt{\frac{R_{g1}}{t_{g1}} + \frac{R_{b1}}{t_{b1}}}}{R_{g1} - R_{b1}}}{\frac{\sqrt{\frac{R_{g2}}{t_{g2}} + \frac{R_{b2}}{t_{b2}}}}{R_{g2} - R_{b2}}} \quad (5.98)$$

Equation (5.98) can be used to compare different counting times in the same system for measuring fixed geometry samples. However, to obtain the best energy window selection in a system, or to compare two systems, the same counting time t should be used:

$$\frac{\sigma_{F1}(R_{S1})}{\sigma_{F2}(R_{S2})} = \frac{\frac{\sqrt{R_{g1} + R_{b1}}}{R_{g1} - R_{b1}}}{\frac{\sqrt{R_{g2} + R_{b2}}}{R_{g2} - R_{b2}}} \quad (5.99)$$

It should be noted that Eqs (5.98) and (5.99) are the same except that in Eq. (5.99) counts are substituted by counting rates.

Equation (5.99) can also be used in planar imaging. Different collimators can be evaluated by comparing counts from a target region to a non-target or background region. However, in imaging, spatial resolution is also important and must be considered.

5.6.7. Estimating required counting times

It is supposed that it is desired to determine the net sample or target counting rate R_s to within a certain fractional uncertainty $\sigma_F(R_s)$. It is supposed further that the approximate gross sample R_{ga} and background R_{ba} counting rates are known from preliminary measurements. If a counting time t is to be used for both the sample or target and the background counting measurements, then the time required to achieve the desired level of statistical reliability is given by Eq. (5.84):

$$t = \frac{R_{ga} + R_{ba}}{[\sigma_F^2(R_s)](R_{ga} - R_{ba})^2}$$

5.6.7.1. Example: calculation of required counting time

The counting time for a thyroid uptake study using a collimated detector is to be determined. The preliminary measurement of the gross thyroid count rate is $R_{ga} = 900$ cpm and background count rate $R_{ba} = 100$ cpm. What counting time is required to determine the net count rate to within 5%?

$$R_{sa} = 900 - 100 = 800 \text{ cpm}$$

$$t = \frac{(900 + 100)}{(0.05)^2 \times (900 - 100)^2} = \frac{1000}{(0.05)^2 \times (800)^2} = 0.625 \text{ min}$$

The time for both the thyroid and background counts is 0.625 min, resulting in a total time of 1.25 min.

5.6.8. Calculating uncertainties in the measurement of plasma volume in patients

A plasma volume (PV) measurement is required on a patient and the uncertainty in the PV measurement is to be calculated. The PV is measured by using the dilution principle. A labelled plasma sample of a known volume is prepared for injection into the patient. A standard sample with the same activity and volume is also prepared for counting. The standard sample is diluted before a sample is counted. Ten minutes after injection of the sample, a blood sample is obtained, the plasma separated from the blood and the blood sample counted. The PV is calculated using the following equation:

$$PV = \frac{R_s}{R_p} VD \tag{5.100}$$

where

Net count rate per millilitre of standard sample $R_s = R_{s+b} - R_b$;

R_b is the count rate of background;

R_{s+b} is the gross count rate per millilitre of standard sample;

Net count rate per millilitre of plasma sample $R_p = R_{p+b} - R_b$;

CHAPTER 5

R_{p+b} is gross count rate per millilitre of plasma sample;
 V is volume of standard sample in millilitres with percentage uncertainty $\sigma_p(V)$;

and D is dilution of standard sample for counting with percentage uncertainty $\sigma_p(D)$.

TABLE 5.7. APPLICATION OF THE PROPAGATION OF ERRORS PRINCIPLE TO THE CALCULATION OF UNCERTAINTIES

Values			Uncertainty in values			
Symbol	Value	Unit	Symbol	Calculation	σ	$\sigma_F(\%)$
t	10	min				
R_{s+b}	3200	cpm	$\sigma(R_{s+b})$	$\sqrt{\frac{R_{s+b}}{t}}$	17.89	0.559
R_b	200	cpm	$\sigma(R_b)$	$\sqrt{\frac{R_b}{t}}$	4.472	2.236
R_s	3000	cpm	$\sigma(R_s)$	$\sqrt{(\sigma(R_{s+b}))^2 + (\sigma(R_b))^2}$	18.44	0.615
R_{p+b}	1200	cpm	$\sigma(R_{p+b})$	$\sqrt{\frac{R_{p+b}}{t}}$	10.95	0.913
R_p	1000	cpm	$\sigma(R_p)$	$\sqrt{(\sigma(R_{p+b}))^2 + (\sigma(R_b))^2}$	11.83	1.183
$\frac{R_s}{R_p}$	3		$\sigma(R_s/R_p)$	$\frac{R_s}{R_p} \sqrt{\left(\frac{\sigma(R_s)}{R_s}\right)^2 + \left(\frac{\sigma(R_p)}{R_p}\right)^2}$	0.040	1.333
V	5	mL	$\sigma(V)$		0.150	3.000
$\frac{R_s V}{R_p}$	15	mL	$\sigma\left(\frac{R_s V}{R_p}\right)$	$\left(\frac{R_s V}{R_p}\right) \sqrt{\left(\frac{\sigma(R_s / R_p)}{R_s / R_p}\right)^2 + \left(\frac{\sigma(V)}{V}\right)^2}$	0.492	3.283
D	200		$\sigma(D)$		6.000	3.000
$PV = \frac{R_s}{R_p} VD$	3000	mL	$\sigma(PV)$	$PV \sqrt{\left(\frac{\sigma((R_s / R_p) V)}{(R_s / R_p) V}\right)^2 + \left(\frac{\sigma(D)}{D}\right)^2}$	133	4.447

The following values were used and measured:

- Counting time $t = 10$ min
- $V \pm \sigma_p(V) = 5 \pm 3\%$ mL
- $D \pm \sigma_p(D) = 200 \pm 3\%$
- $R_{s+b} = 3200$ cpm
- $R_{p+b} = 1200$ cpm
- $R_b = 200$ cpm

The uncertainties are calculated step by step by applying the propagation of errors principle (see Table 5.7)

The measured PV is, therefore, 3000 ± 133 mL or $3000 \pm 4.447\%$. It should be noted that the uncertainty is expressed as one standard deviation. A spreadsheet can be used efficiently to do the calculations in the above table. With a spreadsheet, the influence in changing the counting time or uncertainties in the measurement of the dilution and volume of the standard can be investigated. These spreadsheets are ideally suited for calculations of uncertainties in routine clinical investigations.

5.7. APPLICATION OF STATISTICAL ANALYSIS: DETECTOR PERFORMANCE

5.7.1. Energy resolution of scintillation detectors

We have directed our attention in the previous sections to determine the uncertainty associated with the number of counts measured in a radioactive sample or number of counts in an image pixel. Poisson statistics also play an important role in other aspects of the detection of radiation. A statistical process determines the energy resolution of a detector or the uncertainty associated with the energy measurement of a detected photon. This is the reason why the energy resolution of a solid state detector is significantly better than that of a scintillation detector. The type of detector and the energy of the detected photons determine the energy resolution or uncertainty in the energy of a detected photon. The energy resolution for a detector system and a specific radionuclide does not change from sample to sample. This is different from counting statistics where the uncertainty is determined by the number of counts accumulated during a measurement. Therefore, even for the same sample and same detector system, the uncertainty can change if measurements are repeated following the decay of the nuclide.

Another important consequence of statistics is that in scintillation cameras the location of the position of incoming photons is based on the pulses detected by the detectors. Therefore, the statistics of the detector system limits the spatial resolution that can be achieved with an imaging device. A clear understanding of the statistics associated with the detector when detecting a photon is, therefore, important.

In this section, we will investigate the statistical processes in scintillation detectors, since they are widely used in nuclear medicine for sample counting and imaging.

The operation of scintillation detectors can be considered a three stage process:

- (a) The number x of light photons produced in the scintillator by the detected γ ray;
- (b) The fraction p of the light photons that will eject electrons from the photocathode of the PMT;
- (c) The multiplication M of these electrons multiplied at successive dynodes before being collected at the anode.

The average number N_e of electrons produced at the anode is given by:

$$N_e = xpM \quad (5.101)$$

The fractional variance σ_F^2 in the electron number N for a three stage cascade process is given by Eq. (5.63):

$$\sigma_F^2(N_e) = \sigma_F^2(x) + \sigma_F^2(px) + \sigma_F^2(pxM) \quad (5.102)$$

It can be shown that for dynodes with identical multiplication $\sigma_F^2(M) = \frac{1}{M-1}$. It is assumed that the production of light photons follows a Poisson distribution and, therefore, $\sigma_F^2(x) = \frac{1}{x}$. The fractional variance of the production of electrons from light photons at the photocathode is given by Eq. (5.20) as $\sigma_F^2(p) = \frac{1-p}{p}$:

$$\sigma_F^2(N_e) = \frac{1}{x} + \frac{1}{x} \frac{1-p}{p} + \frac{1}{xp} \frac{1}{(M-1)} \quad (5.103)$$

The fractional energy resolution R_E of detectors is expressed as the FWHM divided by the mean photon energy (Section 5.3.4.1). From Eq. (5.38):

$$R_E = \frac{\text{FWHM}}{E} = \frac{2.355\sigma(E)}{E} = \frac{2.355\sigma(N_e)}{N_e} \quad (5.104)$$

From Eqs (5.103) and (5.104):

$$R_E = 2.355 \sqrt{\frac{1}{x \left(1 + \frac{1-p}{p} + \frac{1}{p(M-1)} \right)}} \quad (5.105)$$

5.7.2. Intervals between successive events

The time intervals separating random events are of interest in nuclear measurements. Such an application is the calculation and measurement of the paralyzable dead time of counting systems.

If r is the average rate at which events are occurring, it follows that $r dt$ is the differential probability that an event will take place in the differential time increment dt . For a radiation detector with unity efficiency, the time interval for counting a single radionuclide is given by:

$$r = \left| \frac{dN}{dt} \right| = \lambda N$$

where

N is the number of radioactive nuclei;

and λ is their decay constant.

In order to derive a distribution function to describe the time interval between adjacent random events, it is first assumed that an event has occurred at time $t = 0$. What is the differential probability that the next event will take place within a differential time dt after a time interval t ?

Two independent processes must take place: no events may occur within the time interval from 0 to t , but an event must take place in the next differential time increment dt . The overall probability will then be given by the product of the probabilities characterizing the two processes, or:

Probability of next event taking place in dt after delay of t	= Probability of number of events during time from 0 to t	×	Probability of event during dt	
$P_1(t) dt$	= $P(0)$	×	$r dt$	(5.106)

The first factor on the right hand side follows directly from the earlier discussion of the Poisson distribution. We seek the possibility that no events will be recorded over an interval of length t for which the average number of recorded events should be rt . From Eq. (5.23):

$$P(0) = \frac{(rt)^0 e^{-rt}}{0!}$$

$$P(0) = e^{-rt} \quad (5.107)$$

Substituting Eq. (5.107) into Eq. (5.106):

$$P_1(t) dt = re^{-rt} dt \quad (5.108)$$

$P_1(t)$ is now the distribution function for intervals between adjacent random events. Figure 5.7 shows the simple exponential shape of the distribution.

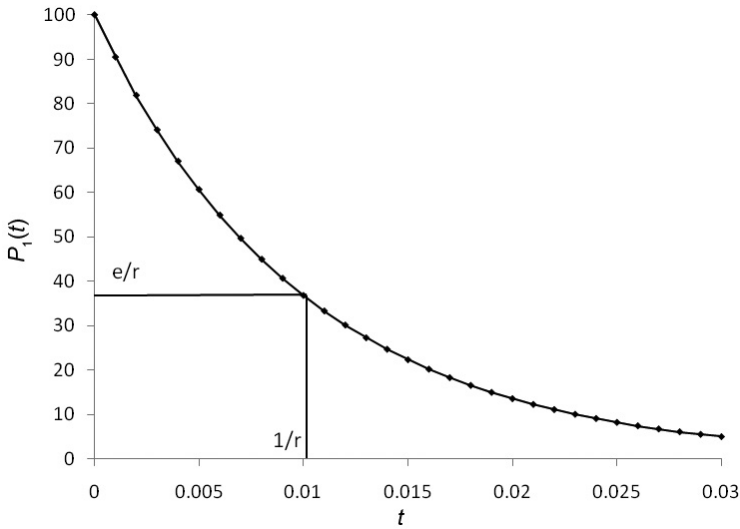


FIG. 5.7. Distribution for intervals between adjacent random events.

It should be noted that the most probable distribution is zero. The average interval length is calculated by applying Eq. (5.8):

$$\bar{t} = \frac{\int_0^{\infty} tP_1(t) dt}{\int_0^{\infty} P_1(t) dt} = \frac{\int_0^{\infty} te^{-rt} dt}{\int_0^{\infty} e^{-rt} dt} = \frac{1}{r} \quad (5.109)$$

5.7.3. Paralyzable dead time

In the paralyzable dead time model, a fixed dead time τ follows each event during the live period of the detector. However, events that occur during the

dead period, although not recorded, still create another fixed dead time τ on the system following the lost event. The recorded rate of events m is identical to the rate of occurrences of time intervals between true events, which exceed τ . The probability of intervals larger than τ can be obtained by integrating Eq. (5.108):

$$P_2(t) dt = \int_{\tau}^{\infty} P_1(t) dt = e^{-r\tau} \quad (5.110)$$

The rate occurrence m of such intervals is obtained by multiplying Eq. (5.110) by the true rate r :

$$m = re^{-r\tau} \quad (5.111)$$

There is no explicit solution for r ; it must be solved iteratively to calculate r from measurements of m and τ . This can be done using a spreadsheet.

BIBLIOGRAPHY

- BUSHBERG, J.T., SEIBERT, J.A., LEIDHOLDT, E.M., BOONE, J.M., The Essential Physics of Medical Imaging, Lippincott Williams and Wilkins, London (2002).
- CHERRY, S.R., SORENSON, J.A., PHELPS, M.E., Physics in Nuclear Medicine, Saunders, Los Angeles, CA (2003).
- DELANEY, C.F.G., FINCH, E.C., Radiation Detectors, Clarendon Press, Oxford (1992).
- KNOLL, G.F., Radiation Detection and Measurement, John Wiley and Sons, New York (1989).
- NATIONAL ELECTRICAL MANUFACTURERS ASSOCIATION, Standards Publication NU 1-2007, Performance Measurements of Gamma Cameras (2007).